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Abstract

To be the lowest bidders in procurement auctions, contractors commonly solicit subcontract bids at the bid preparation stage. In this research, we establish a model of a two-stage procurement auction to analyze such pre-award upstream competitions included in procurement auctions. Our main findings include the following: i) first-price upstream auctions dominate second-price upstream auctions in terms of allocative efficiency and procurement cost; ii) neither a first- nor a second-price auction in upstream competitions implements an optimal outcome; and iii) the ex post negotiation between the prime and subcontractors improves the performance of second-price auctions in upstream competitions. We show that the optimal outcome is implemented with the use of the negotiated second-price auctions in upstream competitions.

Key words: procurement auctions, subcontracting, optimal mechanism
JEL classification: D44, H57, L42

1 Introduction

Subcontracting and outsourcing are common business practices in procurement markets. In a highway construction project, for instance, the winning bidder may subcontract road marking or signal work to specialty firms. In addition, the contractor may purchase raw materials or equipment from other sources, which can also be considered subcontracting in the broader sense. For prime contractors, it is not unusual that the bulk of the cost of a large construction project that covers a wide range of work consists of subcontract payments.

To obtain qualified subcontracts at fair prices, prime contractors (PCs) commonly ask subcontractors (SCs) to bid irrevocable price quotes (subcontract bids) prior to submitting a bid in the procurement auction (Clough and sears (1994), Dyer and Kagel (1996), Marechal and Morand (2003), Grosskopf and Medina (2007)). This practice also satisfies a PC's need to

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obtain the cost estimate for bidding.[1] In this sense, PCs are not only bidders in the downstream procurement auction but also auctioneers in the upstream subcontract auction.

The objective of this paper is to analyze such pre-award upstream auctions included in the procurement auction. We established a model of a two-stage procurement auction in which a procurement buyer auctions off a project for bidders (PCs). In the first stage, each PC solicits subcontract bids from a set of SCs, where the PC is assumed to know the distribution but not the values of the SCs' costs to complete the subcontractable work. If the PC selects an SC, the PC makes a pre-award subcontract agreement with the SC that specifies the amount of the subcontract payment from PC to the winner SC. In the second stage, the PC bids in the procurement auction with private value assumption, given that his cost is the sum of the subcontract payment plus his own cost to complete the remaining non-subcontractable work for himself.

We demonstrate that the use of second-price auctions in upstream competitions results likely in ex post inefficient allocation and a higher procurement cost than the use of first-price auctions. However, the defects of second-price upstream auctions are resolved by introducing an ex post negotiation between the winner PC and sc on the subcontract payment contingent on the outcome in the downstream auction. We show that, with the ex post negotiation, second-price auctions in upstream competitions implement an optimal outcome in procurement auctions with subcontracting.

The closest works related to this research are Hansen (1988) and Wambach (2009), which show the failure of the revenue equivalence in upstream auctions assuming that the outcome of the downstream market is reduced into a downward-sloping demand curve. Due to modeling the downstream auction, in this study, non-trivial extensions are made to their results, including optimal design and efficiency analysis in multi-layered procurement.

The remaining part of this paper is organized as follows. Section 2 describes the model of procurement auctions with pre-award subcontracting. Section 3 examines the equilibrium bidding behavior in upstream auctions. Section 4 analyzes the equilibrium in the downstream auction. Section 5 provides a general model in which PCs are heterogeneous, and Section 6 is the conclusion.

2 The model

Consider a procurement auction in which a procurement buyer solicits bids from N risk-neutral prime contractors (PCs), each denoted by $i \in \{1, \dots, N\}$, to purchase a project. The value of the project to the procurement buyer is equal to V. The procurement buyer sets a reservation price r in the procurement auction so that any bid above r is rejected.

[1] Furthermore, some procurement buyers require PCs to submit a proposed subcontracting plan that must be approved by the contracting officer prior to bidding. For instance, the state of Oregon requires bidders in public projects to submit a list of first-tier subcontractors and their subcontract bids if the amount of the bid is greater than five percent of the total project bid or \$15,000 (ORS 279C.370).

We assume that the work to complete the project consists of two components, the subcontractable and the non-subcontractable work, and the share of the two components is given and identical for all PCs. Prior to submitting a bid in the procurement auction, each PC solicits subcontract bids from n risk-neutral subcontractors (SCs), denoted by $\{\mathrm{SC}_{i,1}, \ldots, \mathrm{SC}_{i,n}\}$.

Prior to being solicited by a PC, each SC draws his cost t to perform the subcontract. The cost t has a commonly known atomless distribution F_t with support $[\underline{t}, \bar{t}]$. The cost of PC to complete the subcontractable work for himself is infinitely large, and PCs always subcontract so that reserve prices in upstream auctions are never below $\bar{t}$. The cost of the PC to complete the remaining non-subcontractable work is normalized to be equal to zero.[2]

The entire game consists of two stages. In the first stage, N upstream auctions occur, and the downstream auction takes place in the second stage. In the first stage, SC (i, j) obtains his cost $t_{i,j}$ and submits a subcontract bid $s_{i,j}$ to PC i in an upstream auction i. An SC will be selected as a winner SC and make a subcontract agreement with PC i, which specifies the amount of a *conditional* subcontract payment c_i from PC i to the SC.[3] In the second stage, PC i submits a bid b_i in the downstream auction with reservation price r given that his cost is equal to c_i.

Throughout this paper, we assume private values. The cost of SC (i, j) is known only to him, and the cost of PC i is known only to PC i. For the regularity condition, $F_t(\cdot)$ satisfies log-concavity *i.e.,* $f_t'(t)/f_t(t)$ is non-increasing.

An optimal outcome in the double-layered procurement can be characterized by considering the situation in which each PC vertically integrates n SCs and bids for the procurement contract. Define $w_i \equiv \min\{t_{i,1}, \ldots, t_{i,n}\}$. The cumulative distribution function of w_i is given by $F_w(w) = 1 - [1 - F_t(w)]^n$. Then, due to Riley and Samuelson (1981) and Myerson (1981), an optimal outcome is identified such that the procurement contract is allocated to the lowest-cost integrated firm whose cost is lower than the reservation price r^* such that $r^* = V - F_w(r^*)/f_w(r^*)$. Taking the optimal outcome as a benchmark, we discuss how it can be implemented through layered procurement auctions.

3 Equilibrium in upstream auctions

If second-price auctions are used in upstream competitions, the subcontract bid only determines the SC's winning probability but not the payoff conditional on winning.[4] Hence, as in the case of standard procurement auctions, submitting $s = t_{i,j}$ is a dominant strategy for the SC in the second-price subcontract auction.[5]

[2]The discussion in which the assumption is relaxed is delivered in Section 5.

[3]The payment c_i is *conditional* because it is paid if the PC i indeed wins in the downstream auction.

[4]In this setting, revenue equivalence holds between an English (ascending) auction and a Vickery (second-price sealed-bid) auction.

[5]Let B denote the lowest competing bid in the upstream auction. By bidding $t_{i,j}$, the bidder will win if $t_{i,j} < B$ and not if $t_{i,j} > B$. Suppose, however, that he bids an amount $t_{i,j} < z$. If $t_{i,j} < z \leq B$, then he still wins in the upstream auction, and his profit is still $t_{i,j} - B$ if the PC wins in the downstream competition. However, if $t_{i,j} < B < z$, then he loses, whereas, if he had bid $t_{i,j}$, he would have made a positive profit

To examine the case of first-price auctions used in upstream competitions, we restrict our attention to a symmetric increasing equilibrium in which all SCs follow an increasing strategy $\sigma(\cdot)$ and all PCs follow an increasing strategy $\beta(\cdot)$. Let $t_{i,(j:n)}$ be the cost of the jth lowest-cost SC solicited by PC i. The cost of PC i is then given by $c_i = \sigma(t_{i,(1:n)})$. To obtain σ, consider the situation in which all SCs other than SC (i,j) follow σ and that in which all PCs follow β. If SC (i,j), $j = 1,\dots,n$, bids s to PC i, SC (i,j) wins the upstream auction with probability:

$$P(s) = [1 - F_t(\sigma^{-1}(s))]^{n-1}.$$

Having his cost c_i equal to s, PC i wins in the downstream auction if and only if c_i is lower than that of all the $N-1$ rivals and reservation price r in the downstream auction. Hence, provided that SC (i,j) wins in upstream auction i, the conditional probability that PC i wins in the downstream competition is given by[6]

$$Q\left(s|N,r\right) = [1 - F_t(\sigma^{-1}(s))]^{n(N-1)}\mathbf{1}_{\{s\le r\}}.$$

SCs receive positive payoffs with probability $P(\cdot)Q(\cdot)$. Hence, the objective function is given by

$$\pi(t_{i,j}|N,r) = \max_s (s - t_{i,j})\left[1 - F_t(\sigma^{-1}(s))\right]^{n(N-1)}.$$

Taking the derivative with respect to s, imposing a symmetric condition, and solving the differential equation yield

$$\sigma(t_{i,j}|N,r) = t_{i,j} + \int_{t_{i,j}}^{r} \left[\frac{1 - F_t(\hat{t})}{1 - F_t(t_{i,j})}\right]^{n(N-1)} d\hat{t}. \tag{1}$$

Let $t_{(1:N)(j:n)}$ be the cost of the jth lowest-cost SC solicited by the winner PC. Then, the expected subcontract price is equal to $E[\sigma(t_{(1:N)(1:n)}|N,r)]$. On the other hand, the expected subcontract price is equal to $E[t_{(1:N)(2:n)}]$ when second-price auctions are used in upstream competitions. Hence, the following lemma is established regarding the expected winning bid in the upstream auction, which extends the results of Hansen (1988) and Wambach (2009) to the model of multi-stage auctions.

Lemma 1. *Subcontract prices are lower when first-price auctions are used than when second-price auctions are used in upstream competitions, namely,*

$$E[\sigma(t_{(1:N)(1:n)}|N,r)] \le E[t_{(1:N)(2:n)}].$$

conditional on the PC's success in the downstream competition. Thus, bidding less than $t_{i,j}$ can never increase his profit but, in some occasions, may in fact decrease it. A similar argument shows that it is not profitable to bid less than $t_{i,j}$.

[6] The decreasing function $Q(s|\cdot)$ is a reminder of the decreasing function $q(\cdot)$ in Hansen (1988) representing the downstream demand schedule.

Proof. Let $t_{(2:N)(j:n)}$ be the cost of the jth lowest-cost SC solicited by the lowest rival PC of the winner in the downstream auction. Equation (1) suggests that, for any $t_{(1:N)(1:n)}$, the subcontract price in the first-price upstream auction

$$\begin{aligned}\sigma(t_{(1:N)(1:n)}|N,r) &= \int_{t_{(1:N)(1:n)}}^{r} \hat{t}n(N-1)\frac{f_t(\hat{t})\left[1-F_t(\hat{t})\right]^{n(N-1)-1}}{\left[1-F_t(t_{(1:N)(1:n)})\right]^{n(N-1)}}d\hat{t}, \\ &= E[\min\{t_{(1:N)(2:n)}, t_{(2:N)(1:n)}, r\}|t_{(1:N)(1:n)}]\end{aligned}$$

never exceeds the conditional expectation of the subcontract price in the second-price upstream auction, $E[t_{(1:N)(2:n)}|t_{(1:N)(1:n)}]$. □

Three observations are particularly noteworthy. First, if first-price auctions are used in upstream auctions, each SC bids as if all other SCs, including the ones bidding for other PCs, were also his rival. Thus, the expected subcontract price declines as N rises or r falls. The equivalence of the expected subcontract price holds only when $N = 1$ and $r \geq \bar{t}$.[7]

Second, PCs using a first-price auction for the upstream competition have stronger bargaining power against SCs as the number of rival PCs increases or the reservation price falls in the downstream auction.[8] This suggests that first-price auctions in upstream competitions allow PCs to share the risk with SCs of uncertainty in the downstream market, while second-price upstream auctions do not.

Finally, the mechanism of the upstream auctions influences the allocative efficiency. In the symmetric equilibrium under the first-price upstream competitions, the lowest-cost SC in the upstream market always receives the subcontract. However, if second-price auctions are used in upstream competitions, the lowest-cost SC misses the subcontract if the cost of his lowest rival SC is higher than the cost of the second-lowest-cost SC in another upstream auction. Hence, the following theorem is established regarding allocative efficiency.

Theorem 1. *The use of a second-price auction in upstream competitions likely leads to an inefficient allocation.*

As a simple example, suppose that $N = 2$, $n = 2$, and $m = 0$ (no common SC). If $t_{1,(1:n)}$ is the lowest-cost SC, possible realizations of SC's costs are

$$\begin{aligned}\mathrm{R}_1 &: t_{1,(1:n)} < t_{1,(2:n)} < t_{2,(1:n)} < t_{2,(2:n)}, \\ \mathrm{R}_2 &: t_{1,(1:n)} < t_{2,(1:n)} < t_{1,(2:n)} < t_{2,(2:n)}, \\ \mathrm{R}_3 &: t_{1,(1:n)} < t_{2,(1:n)} < t_{2,(2:n)} < t_{1,(2:n)}.\end{aligned}$$

[7] If the winner PC solicits SCs after the downstream competition, the outcome is equivalent to the case with $N = 1$ and $r \geq \bar{r}$, in which the equivalence in the expected subcontract price holds. Hence, pre-award subcontracting with the use of first-price auctions dominates the post-award subcontracting conducted with any mechanism from the viewpoint of lowering the subcontract price.

[8] This result holds even if some SCs bid to multiple PCs. As shown in Appendix A, the SC's bidding strategy is identical regardless of whether the SC bids exclusively to a single PC or bids simultaneously to multiple PCs. Thus, although the multiple-bidding SC does not care which PC wins, the SC is induced to bid a lower price by the aggressive bids of exclusively bidding SCs.

Since ts are $i.i.d.$ samples, each of these realizations, $\{\mathrm{R}_1, \mathrm{R}_2, \mathrm{R}_3\}$, occurs with probability 1/3. Thus, if second-price auctions are used in all upstream competitions, an inefficient outcome results with probability 1/3.

When R_3 occurs, the most efficient SC may beneficially negotiate with the PC to undercut the subcontract payment slightly below $t_{2,(2:n)}$. The offer expands the chance of obtaining a positive payoff for the lowest SC but never reduces the PC's payoff. Thus, the use of a second-price auction with negotiation weakly dominates the use of the standard second-price auction for both the PC and the winning SC.

In fact, the ex post negotiation between PCs and SCs resolves the disadvantage of the second-price subcontract auction in revenue and efficiency. Suppose that the downstream contest takes place with a second-price auction. Let b_{-i} be the lowest bid of i's rival PCs in the downstream competition. After the negotiation, the winning PC's subcontract payment will be $\min\{t_{i,(2:n)}, b_{-i}, r\}$. Thus, a weakly dominant strategy of PC i in the downstream auction is to submit $b_i = t_{i,(1:n)}$.[9] Therefore, i's payoff when winning is $\min\{b_{-i}, r\} - t_{i,(2:n)}$ if $\min\{b_{-i}, r\} - t_{i,(2:n)} > 0$ or zero otherwise. Because the amount of the subcontract payment depends on the lowest-rival bid in the downstream competition, the mechanism of the downstream competition must be a second-price auction.

Two observations were then made. First, the negotiated second-price subcontract auction always results in an ex post efficient allocation, since the PC receiving the offer from the lowest-cost SC always wins in the downstream competition. Second, if all PCs use a second-price auction with ex post negotiation, the subcontract price equals the second-lowest cost of all $n \times N$ SCs. Hence, in the winning PC's upstream auction, the expected subcontract price equivalent holds between a first-price and the negotiated second-price auctions.

Since all the benefit from the negotiation belongs to SCs, the negotiated second-price upstream auction may need regulation that restricts the freedom of PCs to choose the mechanism in the upstream market. It follows that cost-minimizing procurement buyers should not entirely delegate subcontracting to PCs.

4 The downstream auction

Let $F_c(\cdot)$ be the distribution of the PC's cost. If a second-price auction is used in upstream auctions, then

$$F_c(c|\mathrm{SP}) = 1 - n(n-1)F_t(c)f_t(c)[1 - F_t(c)]^{n-2}.$$

[9]Since PC i receives a positive payoff if and only if $b_{-i} > t_{i,(2:n)}$, any bid $b \in [t_{i,(1:n)}, t_{i,(2:n)}]$ creates the same expected payoff for PC i.

If first-price auctions are used in upstream competitions and all SCs follow the symmetric equilibrium σ, the PC's cost distribution is given by

$$F_c(\sigma(t|N,r)|N,r,\mathrm{FP}) = 1 - nf_t(t)[1-F_t(t)]^{n-1}. \tag{2}$$

Hence, the distribution is endogenous in N and r only when first-price auctions are used in upstream auctions. Recall that $\sigma(t|N,r)$ declines as N rises or r falls. Since the right-hand side in Equation (2) is constant with respect to N and r, $F_c(\cdot|N,r)$ rises as N rises or r falls. It follows that, if first-price auctions are used in upstream competitions, the distribution of the bidders' private information in the downstream competition shifts to the right in the sense of the first-order stochastic dominance as the number of bidders decreases or the reservation price increases.

The results have impacts on the theoretical and empirical auction models that crucially depend on the assumption that the distribution of private information is independent of the mechanism and the number of agents. For instance, the cost-minimizing reservation price in the downstream auction must be a function of the number of bidders if the upstream contests are conducted through first-price auctions. In addition, the pseudo-values estimated from the structural estimation of auctions must be pooled separately according to the number of bidders to obtain the distribution. For instance, the cost distribution is lower-shifted if the number of bidders in the sample auction is greater than the average. Hence, the estimated cost distribution would be biased upward unless the estimation is conducted separately from that using the pseudo-values obtained from the auction data with a larger number of bidders.

Despite the endogeneity, the distribution of the bidder in the downstream auction is *i.i.d.*. Hence, the downstream auction is revenue-equivalent if all upstream competitions use either a first- or second-price auction. Using the result, we obtained the following theorem regarding the procurement cost.

Theorem 2. *The use of first-price auctions in upstream competitions lowers the procurement cost more significantly than the use of second-price auctions.*

Proof. Let $t_{(2:N)(j:n)}$ be the jth lowest cost of n SCs solicited by the second-lowest PC. The cost of the second-lowest PC is equal to $\sigma(t_{(2:N)(1:n)}|N,r)$ if first-price auctions are used in all upstream auctions, whereas it equals $\min\{t_{(2:N)(2:n)}, r\}$ if second-price auctions are used. By Lemma 1, $\sigma(t_{(2:N)(1:n)}) \leq E[t_{(2:N)(2:N)}]$, which entails that $\sigma(t_{(2:N)(1:n)}|N,r) \leq \min\{t_{(2:N)(2:N)}, r\}$ for any r. Hence, if a second-price auction is used in the downstream auction, the use of first-price auctions in upstream competitions lowers the procurement cost more significantly than the use of second-price auctions. Revenue equivalence in the downstream competition generalizes the result for any mechanism of downstream competition. $\square$

Besides, if the ex post negotiation is possible in all upstream competitions with the form of second-price auctions, the bid of the lowest losing PC is $t_{(2:N)(1:n)}$, whereas it is $\sigma(t_{(2:N)(1:n)})$ if first-price auctions are used in the upstream market. Hence, if the downstream competition is restricted to a second-price auction, the ex post negotiation in a second-price subcontract auction further reduces the procurement cost.

In fact, with the use of a reservation price in the downstream auction, the procurement cost is minimized.

Theorem 3. *Let F_w be the cumulative distribution function of $w_i = \min\{t_{i,1}, \ldots, t_{i,n}\}$. If upstream competitions take place with the use of a negotiated second-price auction, the procurement buyer implements the optimal outcome by using a second-price auction with reservation price r^* such that $r^* = V - F_w(r^*)/f_w(r^*)$ in the downstream competition.*

Proof. The PC wins if the cost of his lowest-cost SC is $t_{(1:N)(1:n)}$ and $t_{(1:N)(1:n)} < r^*$. In addition, the winner PC receives a payment equal to $\min\{t_{(2:N)(1:n)}, r^*\}$. Thus, both allocation and payment rules are identical to the optimal outcome characterized in Section 2. □

The intuitions behind these results are threefold. First, the ex post negotiation between the PC and SC eliminates the double marginalization problem. The expected aggregate informational rents obtained by both winner PC and SC always coincide with the expected payoff of the winning integrated firm in the optimal outcome observed in Section 2. This also implies that the negotiation realizes the optimal risk sharing between the PC and SC on uncertainty of the downstream competition by limiting the informational rent accrued to both PC and SC. Furthermore, the reservation price r^* is independent of N, similar to the case in which PCs vertically integrate SCs. Due to the double marginalization problem, the cost-minimizing reservation price in the case of first-price auctions used in upstream competitions coincides with r^* only when N approaches infinity.

Second, to implement the dominant strategy equilibrium, procurement buyers may need to give an ϵ small amount of transfer to the winner PC so that PCs bid the cost of the lowest SC solicited in the upstream auction. For PC i, submitting any bid between $t_{i,(1:n)}$ and $t_{i,(2:n)}$ yields the same expected profit. The transfer, if any, induces PC i to bid the lowest amount $t_{i,(1:n)}$, in which the winning probability is maximized. As ϵ approaches zero, the expected procurement cost is identical to that in the optimal outcome characterized in Section 2.

Finally, the ex post negotiation requires the downstream competition to be conducted with the second-price auction format. Thus, if a first-price auction must be used in the downstream competition, first-price auctions are the second-best mechanism in upstream competitions from the viewpoint of cost minimization. In this case, the procurement cost falls as the number of PCs increases even if the total number of SCs in the upstream market is fixed.[10] Thus,

[10]Let $Z \equiv n \times N$, and let $t_{(j)}$ be the jth lowest-order statistic among Z *i.i.d.* samples. Suppose also that the downstream competition takes place with a second-price auction. Then, the probability that the procurement cost is equal to or above $\sigma(t_{(j)})$ is $\prod_{k=1}^{j-2}(n-k)/(Z-1)$ for all $j = 3, \ldots, n+1$, and the probability that the procurement cost equals $\sigma(t_2)$ is $(Z-n)/(Z-1)$. Given Z, the probability is increasing in n for any $j = 3, \ldots, n+1$ and decreasing in n for $j = 2$. Hence, the distribution of the procurement cost given n is first-order stochastically dominated by the distribution given $\hat{n} > n$. Since the SC's strategy and their expected payoffs are unchanged, the severer competition in the downstream market extracts more rents only from PCs.

the optimal number of PCs is equal to the number of SCs in the upstream market if the downstream competition uses a first-price auction.

5 The noise term in PC's costs

In this section, we briefly investigate the case in which PCs have heterogeneity in the production cost to complete the non-subcontractable work. Let θ_i be the cost of PC i, $i = 1, \ldots, N$ to complete the work. Then, PC i's total cost c_i is the sum of θ_i plus the subcontract payment. Let $F_\theta(\cdot)$ be the commonly known atomless distribution of θ with support $[\underline{\theta}, \bar{\theta}]$ and let f_θ be its density. As for regularity condition, f_θ is log-concave.

The SC's dominant strategy is still to bid his cost if a second-price auction is used in the upstream competition. On the other hand, suppose that upstream competitions use first-price auctions. Then, if all other SCs follow σ, $P(s) = \left[1 - F_t(\sigma^{-1}(s))\right]^{n-1} \mathbf{1}_{\{s \leq \bar{t}\}}$ is the probability that SC (i, j) will win in upstream auction i when his bid is equal to s. In addition, $Q(s|N, r, \sigma^{-i})$ is the conditional probability that PC i wins in the downstream auction given that SC (i, j) wins in upstream auction i, where σ^{-i} is the strategy profile of $n \times (N-1)$ SCs who bid for PCs except i. Thus, SC (i, j)'s objective function in equilibrium is given as

$$\pi(t_{i,j}|N, r, \sigma^{-i}) = \max_s (s - t_{i,j}) \left[1 - F_t(\sigma^{-1}(s|\cdot)|\cdot)\right]^{n-1} Q(s|N, r, \sigma^{-i}), \tag{3}$$

To obtain $Q(\cdot)$, we first examine the probability that the PC i's total cost c_i is lower than another PC's cost, provided that the PC i's cost for non-subcontractable work equals θ_i and the selected subcontract bid equals s. The convolution theorem gives the cumulative distribution function of c_i by

$$1 - F_c(s + \theta_i|\sigma) = \int_{\underline{t}}^{\bar{t}} n f_t(t) \left[1 - F_t(t)\right]^{n-1} \left[1 - F_\theta(s + \theta_i - \sigma(t|\cdot))\right] dt.$$

Since the number of PCs equals N, the probability that PC i will win in the downstream auction is equal to

$$Q(s|N, r, \sigma^{-i}) = \int_{\underline{\theta}}^{r-s} \left[1 - F_c(s + \theta_i|\sigma)\right]^{N-1} f_\theta(\theta_i) d\theta_i, \tag{4}$$

and the derivative is given by

$$Q'(s|N, r, \sigma^{-i}) = -\int_{\underline{\theta}}^{r-s} \left[1 - F_c(s + \theta_i|\sigma)\right]^{N-1} f'_\theta(\theta_i) d\theta_i \tag{5}$$

if $N > 1$ and $Q' = 0$ if $N = 1$, implying that Q is decreasing in s.

To identify a symmetric increasing equilibrium of the SC's strategy σ, the derivative of (3) is taken with respect to s. Suppressing subscripts and replacing $\sigma^{-1}(s|\cdot) = t$ yield

$$\frac{1}{\sigma(t|N,r)-t} - (n-1)\frac{f_t(t)}{1-F_t(t)}\frac{1}{\sigma'(t|N,r)} = -\frac{Q'(\sigma(t|N,r)|N,r,\sigma^{-i})}{Q(\sigma(t|N,r)|N,r,\sigma^{-i})}. \tag{6}$$

This equation holds for any $\hat{t} \in [\underline{t}, \bar{t}]$ in equilibrium. Thus, the integral is taken from t through $\bar{t}$, and the integration by parts is used on the right-hand side to obtain

$$\sigma(t|N,r) = t + \frac{\int_t^{\bar{t}}[1-F_t(\hat{t})]^{n-1}Q(\sigma(\hat{t}|\cdot)|N,r,\sigma^{-i})d\hat{t}}{[1-F_t(t)]^{n-1}Q(\sigma(t|\cdot)|N,r,\sigma^{-i})}. \tag{7}$$

Solving this for $\sigma(t|\cdot)$ gives the SC's bidding function. Although not a closed form, many insights are drawn from (7). First, for any non-increasing function $Q(\cdot)$, σ is strictly increasing. Second, Lemma 1 still holds: the optimal bidding strategy is to bid the next lowest cost conditional both on the SC's winning in the upstream auction bid and on the PC's winning in the downstream auction.[11] Finally, the SC's bidding strategy σ is a function of the number of PCs and the reservation price in the downstream auction. The hazard function of Q is increasing in N and decreasing in the reservation price r (see Appendix B). Furthermore, the SC's two bidding functions with different numbers of bidders or different reservation prices never cross each other for any $t < \bar{t}$ (see Appendix C). Hence, Lemma 1 holds even if θ is a random variable (see Appendix D).

Theorem 1 can hold when θ is random. For simplicity, we assume that $N = 2$, $n = 2$, and $m = 0$. Suppose that $\sigma(t_{i,(1:n)})$ and θ have the same distribution, namely, $F_\theta(\theta) = 1 - [1 - F_t(\sigma^{-1}(\theta))]^2$. Then, the probability that both $\sigma(t_{1,(1:n)}) < \sigma(t_{2,(1:n)})$ and $\sigma(t_{1,(1:n)}) + \theta_1 > \sigma(t_{2,(1:n)}) + \theta_2$ occur is 0.25,[12] implying that an inefficient allocation occurs with probability 0.25 in first-price upstream auctions. In contrast, if a second-price auction is used in upstream auctions, the probability of an inefficient allocation occurring is strictly more than 0.33. Hence, the advantage of the first-price auction in terms of allocative efficiency is maintained even when unobserved heterogeneity exists in the PC's cost to complete the non-subcontractable work.

[11] Equation (7) is rearranged as

$$\sigma(t|N,r) = \frac{\int_t^{\bar{t}} \hat{t}(n-1)f_t(\hat{t})[1-F_t(\hat{t})]^{n-2}Q(\sigma(\hat{t}|\cdot)|N,r,\sigma^{-i})d\hat{t}}{[1-F_t(t)]^{n-1}Q(\sigma(t|\cdot)|N,r,\sigma^{-i})} + \frac{\int_t^{\bar{t}} \hat{t}[1-F_t(\hat{t})]^{n-1}\left(-Q'(\sigma(\hat{t}|\cdot)|N,r,\sigma^{-i})\right)d\hat{t}}{[1-F_t(t)]^{n-1}Q(\sigma(t|\cdot)|N,r,\sigma^{-i})}.$$

The first term on the right-hand side is the expected cost of the lowest rival SC in the upstream auction conditional on SC's winning and PC i's winning. The second term is the conditional expected cost of PC i's lowest rival PCs.

[12] Let $\mu = t_{1,(1:n)} - t_{2,(1:n)}$, and let $\xi = \theta_1 - \theta_2$. Let $F_\xi(\cdot)$ be the cumulative distribution function for these random variables. By construction, both μ and ξ have a mean of zero, and F_ξ is symmetric so that $F_\xi(0) = 0.5$. For any $\mu \geq 0$, the probability that $\mu + \xi \leq 0$ is $F_\xi(-\mu)$. Hence, if $\mu \geq 0$, then the probability that $\mu + \xi \leq 0$ is

$$\mathcal{P} \equiv \int_0^{\bar{\theta}} F_\xi(-\theta)f_\xi(-\theta)d\theta = \int_{\underline{\theta}}^0 F_\xi(\theta)f_\xi(\theta)d\theta.$$

Integration by parts yields $\mathcal{P} = [F_\xi]_{\underline{\theta}}^0 - \mathcal{P}$. Since $[F_\xi]_{\underline{\theta}}^0 = 0.5$, $\mathcal{P} = 0.25$.

Theorem 2 holds since $c_i = \theta_i + \sigma(t_{i,(1:n)})$ is an *i.i.d.* random variable and the revenue equivalence holds in the downstream auction. As for Theorem 3, define the optimal allocation as the one in which integrated firm i wins if his total cost c_i is the lowest and does not exceed r^* such that $r^* = V - F_c(r^*)/f_c(r^*)$, where $F_c(\cdot)$ is the cumulative distribution function of c_i. Suppose that the downstream competition takes place with a second-price auction and the upstream competitions are on the basis of a second-price auction with the ex post negotiation. Then, the PC will bid $c_i = \theta_i + t_{i,(1:n)}$ and receive $\min\{B_i, r\} - (t_{i(2:n)} + \theta_i)$ if it is positive and zero otherwise. As long as θ_i is known and verifiable by the winner SC, the SC beneficially accepts the payment scheme. Thus, the optimal outcome is implemented.

6 Conclusion

In this study, we have explored the optimal mechanism, efficiency, and equivalence in the procurement cost of a procurement auction with pre-award subcontracting. Motivated by the fact that goods and services are typically produced by a team of firms (main firms and subfirms) in many industries, we constructed a Bayesian game in which the lower-tiered subfirms and suppliers are non-negligible players who also possess private information. Then, we found that, although the aggressive bids of SCs help their PC win if first-price auctions are used in an upstream auction, the double marginalization problem never vanishes. The results show that the use of neither a first- nor a second-price auction in upstream competitions fails to implement the optimal outcome.

The proposed optimal mechanism in upstream competitions suggests that transactions between PCs and SCs in practice may be quite complicated, especially in private projects in which the PC is not selected through the first-price sealed-bid auction. Moreover, the cost-minimizing project owner must take care not only of the mechanism of the downstream market but also of that used in the upstream market, since the auctioneer in the upstream competition, a PC, may not be better off by using the optimal mechanism.

Although our framework may shed new light on some practical questions of subcontracting, there remain many unanswered questions. For instance, the incentive for subcontracting may not stem from cost reduction. Marechal and Morand (2003) point out that subcontracting can reduce the risk of potential change orders.[13] Given the sheer volume of procurement, it is clear that more serious research and evaluation are needed to investigate the effect of subcontracting.

Appendix A

Suppose that there are two types of SCs, *exclusive* and *common.* The exclusive SC submits a subcontract bid to a particular PC, whereas the common SC submits subcontract bids to

[13] The effect of such ex post changes on procurement contracts is thoroughly analyzed in Bajari and Tadelis (2001).

all PCs. Let $m \leq n$ denote the number of common SCs solicited by each PC. In the first stage, SC (l, j) with $l \in \{0, i\}$ obtains his cost $t_{l,j}$ and submits a subcontract bid $s_{l,j}$ to PC i in upstream auction i, and SC $(0, j)$ submits N bids to all PCs. Which SC is common or exclusive is known to PCs. All other settings are the same as those presented in Section 2.

Then, consider a situation in which all SCs other than SC (l, j) follow σ and in which all PCs follow β. If a common SC $(0, j)$, $j = 1, \ldots, m$, bids s to PC i, SC $(0, j)$ wins in the upstream auction with probability $P(s)$, as shown in Section 2. PC i wins in the downstream competition unless any rival PC accepts a subcontract bid below $\sigma(t_{0,j})$. This happens with probability $[1 - F_t(\sigma^{-1}(s))]^{(n-m)(N-1)}\mathbf{1}_{\{s \leq r\}}$. Since all PCs bid the same amount in this occasion, PC i receives the contract with probability $1/N$. Hence, the conditional probability that PC i wins is

$$Q^c(s|N, r) = \frac{1}{N}[1 - F_t(\sigma^{-1}(s))]^{(n-m)(N-1)}\mathbf{1}_{\{s \leq r\}}.$$

The common SC receives a positive payoff with probability $NP(\cdot)Q^c(\cdot)$. Hence, the objective function is identical for both types of SCs.

Appendix B

From (4) and (5),

$$-\frac{Q'(s|N, r, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})} = \frac{\int_{\underline{\theta}}^{r-s}[1 - F_c(s + \theta_i)]^{N-1} f'_\theta(\theta_i) d\theta_i}{\int_{\underline{\theta}}^{r-s}[1 - F_c(s + \theta_i)]^{N-1} f_\theta(\theta_i) d\theta_i}, \tag{8}$$

for any $N > 1$. Let k_0 be a positive real number such that $\frac{f'_\theta(r-s)}{f_\theta(r-s)} = k_0$. Since $\frac{f'_\theta(\theta)}{f_\theta(\theta)}$ is non-increasing in θ, there exists a non-negative and non-increasing function $k(\theta)$ such that, for all $\theta \in [\underline{\theta}, r - s]$, $f'_\theta(\theta)/f_\theta(\theta) - k_0 = k(\theta)$, or equivalently $f'_\theta(\theta) = [k_0 + k(\theta)]f_\theta(\theta)$. Substituting out $f'(\theta)$ in (8) gives

$$\begin{aligned}-\frac{Q'(s|N, r, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})} &= k_0 + \frac{\int_{\underline{\theta}}^{r-s}[1 - F_c(s + \theta_i)]^{N-1} k(\theta_i) f_\theta(\theta_i) d\theta_i}{\int_{\underline{\theta}}^{r-s}[1 - F_c(s + \theta_i)]^{N-1} f_\theta(\theta_i) d\theta_i} \\ &= k_0 + \int_{\underline{\theta}}^{r-s} g(\theta_i|N) k(\theta_i) d\theta_i, \end{aligned} \tag{9}$$

where $g(\theta|N,r) \equiv [1-F_c(s+\theta)]^{N-1} f_\theta(\theta) / \int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i)]^{N-1} f_\theta(\theta_i)d\theta_i$. Define $G(\theta|N,r) = \int_{\underline{\theta}}^{\theta} g(\hat{\theta}|\cdot)d\hat{\theta}$. Then, for any $N > 1$,

$$\begin{aligned}
&G(\theta|N+1) - G(\theta|N) \\
&= \frac{\int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^N f_\theta(\hat{\theta})d\hat{\theta}}{\int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i)]^N f_\theta(\theta_i)d\theta_i} - \frac{\int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^{N-1} f_\theta(\hat{\theta})d\hat{\theta}}{\int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i)]^{N-1} f_\theta(\theta_i)d\theta_i} \\
&= \frac{1}{\Delta}\Bigg\{\int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^N f_\theta(\hat{\theta})d\hat{\theta}\int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i)]^{N-1} f_\theta(\theta_i)d\theta_i \\
&\qquad - \int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^{N-1} f_\theta(\hat{\theta})d\hat{\theta}\int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i)]^N f_\theta(\theta_i)d\theta_i\Bigg\} \\
&= \frac{1}{\Delta}\Bigg\{\int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^N f_\theta(\hat{\theta})d\hat{\theta}\int_{\theta}^{r-s}[1-F_c(s+\theta_i)]^{N-1} f_\theta(\theta_i)d\theta_i \\
&\qquad - \int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^{N-1} f_\theta(\hat{\theta})d\hat{\theta}\int_{\theta}^{r-s}[1-F_c(s+\theta_i)]^N f_\theta(\theta_i)d\theta_i\Bigg\},
\end{aligned}$$

where $\Delta = \int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i)]^N f_\theta(\theta_i)d\theta_i \cdot \int_{\underline{\theta}}^{r-s}[1-F_c(s+\theta_i]^{N+1} f_\theta(\theta_i)d\theta_i > 0$. Applying the Mean Value Theorem yields

$$\begin{aligned}
&= \frac{1}{\Delta}\Bigg\{\big[(1-F_c(\theta^-)) - (1-F_c(\theta^+))\big] \\
&\qquad \times \int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^{N-1} f_\theta(\hat{\theta})d\hat{\theta}\int_{\theta}^{r-s}[1-F_c(s+\tilde{\theta})]^{N-1} f_\theta(\tilde{\theta})d\tilde{\theta}\Bigg\},
\end{aligned}$$

where $\theta^- \in [\underline{\theta}, \theta]$ and $\theta^+ \in [\theta, r-s]$. Since F_c is strictly increasing, the whole terms are strictly positive. Hence, given s, $G(\theta|N+1)$ is first-order stochastically dominated by $G(\theta|N)$. Since $k(\theta_i)$ is non-increasing, $-Q'(s|N,r,\sigma^{-i})/Q(s|N,r,\sigma^{-i})$ is strictly increasing in N.

Finally, we show that $-\frac{Q'(s|N,r,\sigma^{-i})}{Q(s|N,r,\sigma^{-i})}$ is weakly decreasing in r. Let $r < \tilde{r}$. From Equation (9), we obtain

$$\begin{aligned}
&G(\theta|N,\tilde{r}) - G(\theta|N,r) \\
&= \frac{\int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^{N-1} f_\theta(\hat{\theta})d\hat{\theta}}{\int_{\underline{\theta}}^{\tilde{r}-s}[1-F_c(s+\tilde{\theta})]^{N-1} f_\theta(\tilde{\theta})d\tilde{\theta}} - \frac{\int_{\underline{\theta}}^{\theta}[1-F_c(s+\hat{\theta})]^{N-1} f_\theta(\hat{\theta})d\hat{\theta}}{\int_{\underline{\theta}}^{r-s}[1-F_c(s+\tilde{\theta})]^{N-1} f_\theta(\tilde{\theta})d\tilde{\theta}} \\
&\leq 0.
\end{aligned}$$

It follows that $G(\theta|N,\tilde{r})$ first-order stochastically dominates $G(\theta|N,r)$ and that equality holds if and only if $F_c(r) = 1$. Hence, from Equation (9), we obtain

$$\begin{aligned}
&-\frac{Q'(s|N,\tilde{r})}{Q(s|N,\tilde{r})} - \left(-\frac{Q'(s|N,r)}{Q(s|N,r)}\right) \\
&= \int_{\underline{\theta}}^{\tilde{r}-s} g(\theta_i|N,\tilde{r})k(\theta_i)d\theta_i - \int_{\underline{\theta}}^{r-s} g(\theta_i|N,r)k(\theta_i)d\theta_i \\
&= \int_{\underline{\theta}}^{r-s} \left[g(\theta_i|N,\tilde{r}) - g(\theta_i|N,r)\right] k(\theta_i)d\theta_i + \int_{r-s}^{\tilde{r}-s} g(\theta_i|N,\tilde{r})k(\theta_i)d\theta_i \\
&= k(\theta^-)\int_{\underline{\theta}}^{r-s} \left[g(\theta_i|N,\tilde{r}) - g(\theta_i|N,r)\right] d\theta_i + k(\theta^+)\int_{r-s}^{\tilde{r}-s} g(\theta_i|N,\tilde{r})d\theta_i \\
&\leq 0,
\end{aligned}$$

where $\theta^- < \theta^+$. The third equality is obtained by the Mean Value Theorem. Furthermore, $\int_{\underline{\theta}}^{\tilde{r}-s} g(\cdot|N,\tilde{r}) = \int_{\underline{\theta}}^{r-s} g(\cdot|N,r) = 1$, or equivalently

$$\int_{\underline{\theta}}^{r-s} \left[g(\theta|N,\tilde{r}) - g(\theta|N,r)\right] d\theta + \int_{\underline{\theta}}^{\tilde{r}-s} g(\theta|N,\tilde{r})d\theta = 0.$$

Together with the fact that $k(\cdot)$ is non-increasing, we obtain the last inequality.

Appendix C

Suppose there exists t such that $\sigma(t|\tilde{m},\cdot) = \sigma(t|m,\cdot) = \xi(t)$. Then, by Equation (7),

$$\int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|\tilde{m},r,\sigma^{-i})}{Q(\xi(t)|\tilde{m},r,\sigma^{-i})} d\hat{t} = \int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|m,r,\sigma^{-i})}{Q(\xi(t)|m,r,\sigma^{-i})} d\hat{t}.$$

Since $-\frac{Q'(s|m,r,\sigma^{-i})}{Q(s|m,r,\sigma^{-i})}$ is increasing in m,

$$\begin{aligned}
0 &> \frac{Q(s|\tilde{m},r,\sigma^{-i})}{Q(s|m,r,\sigma^{-i})} \left\{ \frac{Q'(s|\tilde{m},r,\sigma^{-i})}{Q(s|\tilde{m},r,\sigma^{-i})} - \frac{Q'(s|m,r,\sigma^{-i})}{Q(s|m,r,\sigma^{-i})} \right\} \\
&= \frac{Q'(s|\tilde{m},r,\sigma^{-i})Q(s|m,r,\sigma^{-i}) - Q'(s|m,r,\sigma^{-i})Q(s|\tilde{m},r,\sigma^{-i})}{[Q(s|m,r,\sigma^{-i})]^2} = \frac{d}{ds}\frac{Q(s|\tilde{m},r,\sigma^{-i})}{Q(s|m,r,\sigma^{-i})},
\end{aligned}$$

for any $\tilde{m} > m \geq 1$. It follows that, for any $\hat{s} > s$,

$$\frac{Q(\hat{s}|\tilde{m},r,\sigma^{-i})}{Q(\hat{s}|m,r,\sigma^{-i})} < \frac{Q(s|\tilde{m},r,\sigma^{-i})}{Q(s|m,r,\sigma^{-i})} \Leftrightarrow \frac{Q(\hat{s}|\tilde{m},r,\sigma^{-i})}{Q(s|\tilde{m},r,\sigma^{-i})} < \frac{Q(\hat{s}|m,r,\sigma^{-i})}{Q(s|m,r,\sigma^{-i})}.$$

Replacing $\hat{s} = \xi(\hat{t})$ and $\xi = \sigma(t)$ and multiplying by $[1 - F_{t(\hat{t})}]^{n-1}$ on both sides will yield

$$[1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|\tilde{m}, r, \sigma^{-i})}{Q(\xi(t)|\tilde{m}, r, \sigma^{-i})} < [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|m, r, \sigma^{-i})}{Q(\xi(t)|m, r, \sigma^{-i})}$$
$$\Leftrightarrow \int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|\tilde{m}, r, \sigma^{-i})}{Q(\xi(t)|\tilde{m}, r, \sigma^{-i})} d\hat{t} < \int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|m, r, \sigma^{-i})}{Q(\xi(t)|m, r, \sigma^{-i})} d\hat{t}.$$

Thus, we obtain a contradiction.

Next, suppose by contradiction that there exists t such that $\sigma(t|N, \tilde{r}) = \sigma(t|N, r) = \xi(t)$. Then, by Equation (7),

$$\int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|N, \tilde{r}, \sigma^{-i})}{Q(\xi(t)|N, \tilde{r}, \sigma^{-i})} d\hat{t} = \int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|N, r, \sigma^{-i})}{Q(\xi(t)|N, r, \sigma^{-i})} d\hat{t}.$$

Since $-\frac{Q'(s|N,r,\sigma^{-i})}{Q(s|N,r,\sigma^{-i})}$ is decreasing in r,

$$0 < \frac{Q(s|N, \tilde{r}, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})} \left\{ \frac{Q'(s|N, \tilde{r}, \sigma^{-i})}{Q(s|N, \tilde{r}, \sigma^{-i})} - \frac{Q'(s|N, r, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})} \right\} = \frac{d}{ds} \frac{Q(s|N, \tilde{r}, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})}$$

for any $r < \tilde{r}$. It follows that, for any $\hat{s} > s$,

$$\frac{Q(s|N, \tilde{r}, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})} < \frac{Q(\hat{s}|N, \tilde{r}, \sigma^{-i})}{Q(\hat{s}|N, r, \sigma^{-i})} \Leftrightarrow \frac{Q(\hat{s}|N, r, \sigma^{-i})}{Q(s|N, r, \sigma^{-i})} < \frac{Q(\hat{s}|N, \tilde{r}, \sigma^{-i})}{Q(s|N, \tilde{r}, \sigma^{-i})}.$$

Replacing $\hat{s} = \xi(\hat{t})$ and $\xi = \sigma(t)$ and multiplying by $[1 - F_{t(\hat{t})}]^{n-1}$ on both sides will yield

$$[1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|N, r, \sigma^{-i})}{Q(\xi(t)|N, r, \sigma^{-i})} < [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|N, \tilde{r}, \sigma^{-i})}{Q(\xi(t)|N, \tilde{r}, \sigma^{-i})}$$
$$\Leftrightarrow \int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|N, r, \sigma^{-i})}{Q(\xi(t)|N, r, \sigma^{-i})} d\hat{t} < \int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} \frac{Q(\xi(\hat{t})|N, \tilde{r}, \sigma^{-i})}{Q(\xi(t)|N, \tilde{r}, \sigma^{-i})} d\hat{t}.$$

Thus, we have reached a contradiction.

Appendix D

Since $\sigma(t|m)$ and $\sigma(t|\tilde{m})$ never cross each other at $t < \bar{t}$ for any $\tilde{m} > m \geq 1$, $\sigma(t|m)$ must be monotone in m, the number of bidders, even if m is a real number instead of an integer. Suppose by contradiction that $\sigma(t|m)$ is increasing in $m \geq 1$. Then, replacing N with a real number m in (6) yields

$$\frac{1}{\sigma(t|m, r) - t} - (n-1) \frac{f_t(t)}{1 - F_t(t)} \frac{1}{\sigma'(t|m, r)} = -\frac{Q'(\sigma(t|m, r)|m, r, \sigma^{-i})}{Q(\sigma(t|m, r)|m, r, \sigma^{-i})}.$$

The right-hand side is positive if $m > 1$ and vanishes if $m = 1$. Therefore, if the number of bidders increases to $\tilde{m} > 1$, then $\sigma'(t|\tilde{m}, \cdot) > \sigma'(t|m, \cdot)$ must hold for all $t \in [\underline{t}, \bar{t})$. Thus,

$\int_{t}^{\bar{t}} \sigma'(\hat{t}|\tilde{m}, \cdot) d\hat{t} = \bar{t} - \sigma(t|\tilde{m}, \cdot) > \bar{t} - \sigma(t|m, \cdot) = \int_{t}^{\bar{t}} \sigma'(\hat{t}|\tilde{m}, \cdot) d\hat{t}$, implying $\sigma(t|\tilde{m}, \cdot) < \sigma(t|m, \cdot)$. A contradiction is reached.

Similarly, $\sigma(t|N, r)$ is strictly increasing in r. Letting $r^- \leq \underline{t} + \underline{\theta} < \bar{t} + \underline{\theta} \leq r^+$, suppose by contradiction that there exists $t < \bar{t}$ such that $\sigma(t|N, r^+) < \sigma(t|N, r^-)$. Since an SC has no chance to obtain a job if the reservation price in the downstream auction is equal to r^-, his strategy is $\sigma(t|\cdot, r^-) = t$ for all t, whereas if $r = r^+$, then an SC seeks a positive bid margin when his cost is strictly smaller than $\bar{t}$, *i.e.*, $\sigma(t|\cdot, r^+) > t$ for any $t < \bar{t}$. A contradiction is reached. Thus, $\sigma(t|N, r)$ is strictly increasing in r.

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