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Pareto-improving Immigration in the Presence of Social Security

by

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Pareto-improving Immigration in the Presence of Social Security

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Abstract

The effect of accepting more immigrants on welfare in the presence of a pay-as-you-go social security system is analyzed theoretically and quantitatively. First, it is shown that if initially there exist intergenerational government transfers from the young to the old, the government can lead an economy to the (modified) golden rule level within a finite time in a Pareto-improving way by increasing the percentage of immigrants to natives (PITN). Second, using the computational overlapping generation model, I calculate both the welfare gain of increasing the PITN from 15.5 percent to 25.5 percent and years needed to reach the (modified) golden rule level in a Pareto-improving way in a model economy. The simulation shows that the present value of the Pareto-improving welfare gain of increasing the PITN comprises 23 percent of the initial GDP.

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It takes 112 years for the model economy to reach the golden rule level in a Pareto-improving way.

1 Introduction

Transforming a pay-as-you-go (PYGO) social security system into a funded system is not easy. When the PYGO social security system is changed to a funded system, some generations must bear the so called “the double burden”, such that a young generation needs to pay the social security tax twice. Thus, although the transition from a PYGO social security to a funded system is desirable since a PYGO social security system causes under-accumulation of capital, it is difficult to transit in a Pareto-improving way.

On the other hand, in the face of the fiscal problems caused by negative demographic shocks in the presence of PYGO social security, policy makers have recently shown increased interests in accepting more immigrants. For example, the IMF report on the Japanese economy (2012), which is suffering as a result of an extremely low fertility rate and practical restrictions on the inflow of immigrants, states that facilitating a modest level of immigration could have a large payoff for the Japanese economy. However, although using immigrants to solve the fiscal problem of PYGO social security in the short run is attractive, its desirability in the long run is unclear. Increasing the percentage of immigrants to natives implies a higher population growth rate in a host country. With a neoclassical production function that exhibits the diminishing marginal product of capital, the standard growth model (Ramsey (1928) and Solow (1964)) predicts that such a higher population growth rate leads to both a lower level of capital stock per capita and income per capita, starting from a dynamically efficient initial steady state. Thus, it is not clear how such an increase of immigrants can help to solve the problem of PYGO social security in the long run. The literature also presents an unclear picture. In the literature on public finance, there is increasing interest in the effect of accepting more immigrants on the social welfare. By using the computational overlapping generation model (Auerbach and Kotlikoff model(Auerbach and Kotlikoff (1998))), Storesletten (2000) argues that accepting a particular type of immigrants, (skilled who are of an age such that they will not be able to claim the social security benefit because they will not be able to satisfy the

minimum requirement of the duration of social security tax payments) will increase the social welfare in the presence of the retirement of the baby boom generation. In contrast, Fehr, Jokisch and Kotlikoff (2004) argue that there will be no such a welfare gain. Feldstein (2006) analyzes the effect of immigration in Spain and concludes that immigration does not bring a welfare gain while Collado, Iturbe-Ormaetxe and Valera (2004) argue that accepting more immigrants brings a positive welfare gain to Spain.

Given the mixed results in the literature regarding the effect of accepting more immigrants on the social welfare and the theoretical prediction of the standard growth model, a natural question arises whether or not, from a theoretical standpoint, accepting more immigrants Pareto-improves welfare in the presence of PYGO social security. With a neoclassical production function that exhibits diminishing marginal product of capital, a decrease of income per capita at the steady state seems inevitable when the economy experiences a higher population growth rate due to an increased inflow of immigrants. Yet, if this is so, one may wonder why the previous studies have arrived at such different results regarding the effect on welfare of accepting more immigrants.

Motivated by those questions, I analyze theoretically and qualitatively the effect on welfare of accepting more immigrants and increasing the population growth rate. I conclude that accepting more immigrants and increasing the population growth rate Pareto-improves welfare and, to a large extent, solves in a Pareto-improving way the problem of under-accumulation of capital that is caused by implementing a PYGO social security system.¹ More specifically, firstly, using the overlapping generation model developed by Diamond (1965), I show that in an economy with or without distorting taxes it is Pareto-improving to increase the percentage of immigrants to natives (PITN) if there exist upward intergenerational transfers, in the sense that the marginal product of labor of a young individual times labor supply is greater than the sum of resources that a young individual consumes when he or she is young and the

¹The Matlab code which is used for this simulation is available from the Journal' website and from the author.

amount of resources that are transferred to future periods(the MPL condition). Note that the former is the pre-tax income of a young individual and the latter is equal to the sum of the after tax income, the amount of publicly provided private goods for a young individual and the government saving per each young individual. In the presence of a PYGO social security system, some of the pre-tax income of a young individual is used for the consumption of the old. Thus, the above MPL condition is likely to be satisfied unless the government have a large amount of the government savings. Thus, in the presence of PYGO social security, this MPL condition is likely to be satisfied. Secondly, I show analytically that when this MPL condition is satisfied, the government can lead the economy to the (modified) golden rule level in a Pareto-improving way within a finite time by putting in savings the government budget surplus, which is obtained by increasing the PITN. Note that when the economy reaches the golden rule level, the problem of under-accumulation of capital caused by the PYGO social security is solved for all practical purposes. Third, I quantify this Pareto-improving welfare gain that is yielded by increasing the PITN in the presence of a PYGO social security system and calculate the year needed to reach the (modified) golden rule level in a Pareto-improving way by using the computational overlapping generation model developed by Auerbach and Kotlikoff (1987). I consider a moderate increase of the PITN, such that the PITN starts to increase from 15.5 percent, reaches 25.5 percent at the 80th year and remains constant at 25.5 percent in later years.² With this speed of increase of the PITN and in the model that mimics important dimensions of the US economy, my simulation shows it takes a minimum of 112 years for the model economy to reach the golden rule level in a Pareto-improving way. On the new balanced growth path, the capital stock per efficient unit of labor increases by 102 percent and the publicly provided private goods per capita increases by 36 percent. When the target capital stock is set at the modified golden rule level with 3 percent of intergenerational discount rate, it takes 65 years to reach the modified golden rule level in a Pareto-improving way

²15.5 percent initial PITN is obtained by using census 2000 data from the author's calculation. See section 4.2 for more detailed discussion.

and the capital stock per efficient unit of labor increases by 18 percent. The present discounted value (PDV) of the Pareto-improved utility, measured by the expenditure function, of natives and their descendants, which does not include the increased utility of immigrants and their descendants, comprises 23 percent of the initial GDP. When the time to reach the target PITN is shortened to 42 years, the economy reaches the modified golden rule at the 59th year and the PDV of the Pareto improvement comprises 28 percent of the initial GDP. Finally, I conduct robustness checks by changing a number of parameter values, for example, the share of the surplus for the government savings, the replacement rate, the time preference rate, the risk aversion, the initial government debt(asset) level, the level of immigrants' earnings and the consumption of public services by immigrants. Those robustness checks show that the results of the simulation do not change substantially in magnitude for different parameter values. Both theoretical results and the computational results suggest the robustness of the welfare gain of increasing the PITN in the presence of PYGO social security.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents a theoretical analysis. Section 4 presents a simulation-based analysis using the computational overlapping generation model. Section 5 concludes.

2 Literature

In the theoretical literature on the effect of accepting immigrants, Razin (1999) is, to the best of my knowledge, the first paper that shows that accepting immigrants improves welfare. He shows that in a small open economy model in which factor prices are fixed, accepting more immigrants can improve welfare in the presence of PYGO social security. However, in a subsequent paper, Razin and Sadka (2000) show that in an closed economy model in which capital accumulation and factor prices are endogenous, the result obtained under the assumption of a small open economy is not likely to hold.

For the empirical side, there is a large volume of literature that analyzes the effect of accepting immigrants. (Huddle(1993), Borjas(1994), Passel (1994), Simon (1984) and Akbari (1989), Lee and Miller (1997), Auerbach and Preopoulous (1999), Storesletten (2003), and, Collado and Valera (2004)). More recently, this literature analyzes the effect of accepting immigrants in a dynamic general equilibrium model (Storesletten(1995, 2000), Canova and Ravn (2000), and Fehr, Joisch and Kotlikoff (2004)). Storesletten (1995) is the first paper to analyze the effect of accepting immigrants in a dynamic general equilibrium mode with aggregate uncertainty and Storesletten (2000) analyzes without aggregate uncertainty. He argues, in the US context, that by selecting a particular type of immigrants, the acceptance of immigrants can have a positive effect on the welfare of the native. Fehr, Joisch and Kotlikoff (2004) argues that there is no such welfare gain, or, even if there were to be a gain, it would be very small.

This paper contributes to the existing literature in several ways. First, I identify the conditions under which it is Pareto-improving to accept more immigrants. (MPL condition). Second, I show that the government can lead the economy to the (modified) golden rule level by accepting more immigrants within a finite time in a Pareto-improving way. This is in sharp contrast to the literature on the social security reform, in which it is argued that it is difficult to increase the capital stock through social security reform in a Pareto-improving way (Geanakoplos, Mitchell and Zeldes (1998)). Third, I develop the method that allows me to show that accepting more immigrants Pareto-improves all generations in the presence of distorting taxes and changing factor prices in a dynamic economy. This technique was originally developed in the international trade literature to show the superiority of free trade over restricted trade in the absence of lump-sum taxes and transfers (Dixit and Norman (1980)). To the best of my knowledge, this is the first paper that applies this technique to the analysis of a dynamic economy. Fourth, I quantify the welfare gain of this Pareto-improvement, which is predicted by the theoretical model. Consistent with the results of Storesletten(2000), I show that there is a non-trivial welfare gain of increasing immigrants in the US and demonstrate that my results are robust for

different values of the parameters. The theoretical results and the robustness of the simulation results show that a policy of increasing immigrants in the US can be an important policy option to be considered.

3 The model

The model uses the standard overlapping generation model with a neoclassical production function developed by Diamond (1965). Each individual lives for two periods. When individuals are in the first period, they work and are called “young”. When they are in the second period, they are retired and are called “old”. I assume that immigrants come to the host country only when they are young and that the government of the host country prohibits the immigrants from immigrating when they are already old. I define individuals who are born at the beginning of period t in the host country as natives of cohort t , regardless of the nationality of the parents. Immigrants who move to the host country at the beginning of period t are considered as immigrants of cohort t . Let j be the index indicating nationality. If an individual is a native, $j = n$ and if she or he is an immigrant, $j = m$. Let N_t^j be the number of the young of type j in period t . Let $(c_t^{y,j}, c_{t+1}^{o,j})$ be the consumption in the young period and the old period of a type j ($j = n, m$) individual of cohort t . Let $g_t^{y,j}$ be the amount of publicly provided private goods, such as education and government provided health care service for the young, for each young individual of type j which is consumed at period t . Let $g_t^{o,j}$ be the amount of publicly provided private goods, such as medicaid and publicly provided nursing home, for each old individual of type j at period t . Let $g_t^{ind,j}$ be the amount of age-independent publicly provided private goods that are consumed by a young individual and an old individual of type j .³ I

³In this paper, I ignore non-rivalry public goods. Note that the presence of non-rivalry public goods will favor immigration because accepting immigrants means that the cost of non-rivalry public goods will be shared by more individuals without decreasing their consumption.

assume that the utility function of cohort of type j is

$$U^j(c_t^{yj}, l_t^j, g_t^{yj}, g_t^{ind}, c_{t+1}^{oj}, g_{t+1}^{oj}, g_{t+1}^{ind,j}) = u^{yj}(c_t^{yj}, l_t^j) + v^{yj}(g_t^{yj}, g_t^{ind,j}) + \frac{1}{1+\rho}[u^{oj}(c_{t+1}^{oj}) + v^{oj}(g_{t+1}^{oj}, g_{t+1}^{ind,j})] . \quad (1)$$

I assume that $u^{ij}(c_t^{ij}, l_t^j)$ and $v^i(g_t^{ij}, g_t^{ind,j})$ ($i = y, o; j = n, m$) are strictly increasing and concave functions. I assume additive separability of publicly provided private goods so that the provision of publicly provided private goods does not affect the consumption and saving decisions of individuals. This assumption simplifies the analysis because I also assume that the government redistributes the welfare gain of accepting more immigrants in the form of increased publicly provided private goods to individuals.⁴

For the production side, let $F(L_t, K_t)$ be a production function where L_t and K_t are the total amount of labor and the total capital stock used at period t . Let δ be the depreciation rate of the capital. I assume that $F(L_t, K_t)$ exhibits constant returns to scale and that both the marginal product of labor and capital are diminishing. I assume that the standard Inada condition is satisfied.

I assume that the economy is at the steady state initially and that the initial economy is dynamically efficient.⁵ Furthermore, for the welfare analysis of accepting more immigrants, I make the following additional assumptions.

AS1: The amount of publicly provided private goods per person, $(g_t^{yj}, g_t^{oj}, g_t^{ind,j})$ is constant at the initial steady state and $(g_t^{yj}, g_t^{oj}, g_t^{ind,j}) = (g^{y,j}, g^{o,j}, g^{ind,j})$.

AS2: The government uses a PYGO social security system at the initial steady state.

AS3: For one unit supply of labor by a native, ϕ^n efficient units of labor is supplied.

For one unit supply of labor by an immigrant, ϕ^m efficient units of labor is supplied where $\phi^m \leq \phi^n$. For normalization, I assume that $\phi^n = 1$.

⁴There are several ways to redistribute the welfare gain to individuals. I use this method to simplify the analysis. The main conclusion does not change when other way of redistributing the welfare gain are used.

⁵In the literature, it is well-known that if the market interest rate is lower than the population growth rate, it is possible to Pareto-improve welfare (dynamic inefficiency). Since this paper's interest is not such a dynamic inefficiency problem, I postulate that at the initial steady state, the market interest rate is higher than the population growth rate (Cass(1972)).

AS4: The descendants of immigrants integrate with the native population and earn the same income as natives.

AS5: Immigrants and their children stay permanently in the host country.

AS6: The fertility rate of immigrants is equal to or higher than the fertility of the native.

AS7: If immigrants and natives have the same productivities, then the government treats immigrants and native in the same way in the tax and social security system.

I use AS1 to focus on the issue of immigration, rather than on issues of public expenditure. I need AS2 to see the effect of increasing the number of immigrants in the presence of a pay-as-you go social security. AS7 needs more discussion. Clearly, if the government can treat the immigrants in a discriminating way in the tax and the public pension systems, there is a way to increase the utility of both natives and immigrants. Normally, the wage rate of the immigrants in their country of origin is lower than the wage rate in their host country. Thus, it is possible to Pareto-improve the welfare of both natives and immigrants if (a) the government in the host country sets a high tax rate on incoming immigrants in such a way that the net wage rate of immigrants in their host country is still higher than their net wage rate of their country of origin, (b) the government redistributes to natives the tax revenue collected from immigrants. AS4 precludes such an obvious case from occurring.

Let F_K and F_L be the partial derivative of the production function with respect to capital and labor. Let w_t^j and r_t be the wage rate of an individual type j and the interest rate at period t . Let s_t^j and a_t be the amount of savings made by cohort t of type j and the total amount of the government savings divided by the number of cohort t . Then, w_t^j and r_t are determined as follows:

$$w_t^n = F_L(L_t, K_t), w_t^m = \phi^m F_L(L_t, K_t), r_t = F_K(L_t, K_t) - \delta$$

$$\text{where } L_t = \sum_j \phi^j l_t^j N_t^j \text{ and } K_t = \sum s_{t-1}^j N_{t-1}^j + a_{t-1} \sum N_{t-1}^j \quad (2)$$

The resource constraint at the period t is as follows:

$$\begin{aligned} & F(L_t, K_t) + (1 - \delta)K_t \\ &= \sum \{c_t^{y,j} + s_t^j + g^{y,j} + g^{ind,j} + a_t\} N_t^j + \sum N_{t-1}^j \{c_t^{o,j} + g^{o,j} + g^{ind,j}\} \end{aligned} \quad (3)$$

Let the fertility rate of natives and immigrants be π_n and π_m , respectively. Given π_n and π_m , N_t^n can be written as follows:

$$N_t^n = (1 + \pi_m) \times N_{t-1}^m + (1 + \pi_n) \times N_{t-1}^n \quad (4)$$

Let α_t be the immigrant-to-native ratio (INR) of cohort t . The immigration policy is expressed in terms of α_t . For example, a one-time increase of INR means that $\alpha_0 = \alpha^*$, $\alpha_1 = \tilde{\alpha}$ and $\alpha_t = \alpha^*$ for $t \geq 2$ where $\tilde{\alpha} > \alpha^*$. Permanently increasing INR means that $\alpha_0 = \alpha^*$ and $\alpha_t = \tilde{\alpha}$ for all $t \geq 1$ where $\tilde{\alpha} > \alpha^*$. The sum of natives and immigrants of cohort t is

$$\begin{aligned} \sum_{j=n,m} N_t^j &= N_t^n \times (1 + \alpha_t) \\ &= \{(1 + \pi_m) \times N_{t-1}^m + (1 + \pi_n) \times N_{t-1}^n\} \times (1 + \alpha_t) \\ &= \{(1 + \pi_m) \times \alpha_{t-1} + (1 + \pi_n)\} \times N_{t-1}^n (1 + \alpha_t) \end{aligned} \quad (5)$$

The total number of cohort $t - 1$ is $N_{t-1}^n (1 + \alpha_{t-1})$. Thus, at the steady state immigration policy α^* , the growth rate of cohorts is $(1 + \pi_m) \times \alpha^* + (1 + \pi_n) - 1$. Define $R(\alpha)$ as follows:

$$R(\alpha) \equiv (1 + \pi_m) \times \alpha + (1 + \pi_n)$$

We can interpret $R(\alpha)$ as one plus the cohort population growth rate when the immigration policy α is implemented. To avoid a situation in which the total population of period t becomes zero or negative, I assume

$$R(\alpha_t) \equiv (1 + \pi_m) \times \alpha_{t-1} + (1 + \pi_n) > 0 \text{ for } \alpha_{t-1} \geq 0. \quad (6)$$

3.1 An Economy with Lump-sum tax, Same Productivities and Preferences: Deriving MPL condition

The purpose of the section is to derive the MPL condition which plays an important role in the analysis of an economy with distorting taxes and in a quantitative analysis. To obtain MPL condition in a clear way, first I consider an economy where the lump-sum tax is available for the government and immigrants and natives have same productivities and preferences. This implies that there are no distorting taxes. Once we get the MPL condition in this economy, I will show that this MPL condition plays the critical role for evaluating the effect of increasing the number of immigrants on welfare and capital accumulation in an economy with distorting taxes and with different productivities and preferences between immigrants and natives. Readers who are interested in the analysis of the economy where immigrants and native are different in terms of productivities, consumption of public services and preferences can skip this section and go to sub-section 3.4 directly.

Let b_{t+1}^j be the social security benefit for the cohort t at the period $t + 1$. Let τ_t^j be the amount of the lump-sum tax at the period t for type j . The assumption of the same productivities and the same preference between natives and immigrants and AS7 imply that $\phi^j, w_t^j, c_t^{yj}, c_{t+1}^{oj}, l_t^j, s_t^j, b_{t+1}^j, \tau_t^j$ do not change for different value of j . Thus, we eliminate superscript j for those variables and from the utility functions. Cohort t maximizes the life-time utility function subject to the budget constraint. The budget constraint of the cohort t of type j is

$$w_t l_t - \tau_t = c_t^y + s_t \text{ and } b_{t+1} + (1 + r_{t+1})s_t = c_{t+1}^o . \quad (7)$$

The government budget constraint at period t is

$$(\tau_t - g^y - g^{ind} - a_t) \sum_{j=n,m} N_t^j - (b_t + g^o + g^{ind} - (1 + r_t)a_{t-1}) \sum_{j=n,m} N_{t-1}^j = 0$$

Using the individual budget constraint, the homogeneity of the production function and equation (2), it is straightforward to show that the government budget

constraint is equivalent to the following resource constraint:⁶

$$\begin{aligned}
& F(L_t, K_t) + (1 - \delta)K_t \geq \\
& \{c_t^y + s_t + g^y + g^{ind} + a_t\} \sum_{j=n,m} N_t^j + \{c_t^o + g^o + g^{ind}\} \sum_{j=n,m} N_{t-1}^j \\
& \text{where } L_t = \sum_{j=n,m} l_t N_t^j \text{ and } K_t = (s_{t-1} + a_{t-1}) \sum_{j=n,m} N_{t-1}^j
\end{aligned} \tag{8}$$

Using (5), the above resource constraint can be rewritten as

$$\begin{aligned}
& F(L_t, K_t) + (1 - \delta)K_t \geq \\
& \{c_t^y + s_t + g^y + g^{ind} + a_t\} N_{t-1}^n R(\alpha_{t-1})(1 + \alpha_t) + \{c_t^o + g^o + g^{ind}\} N_{t-1}^n (1 + \alpha_{t-1})
\end{aligned}$$

Before analyzing the effect of accepting more immigrants, we characterize the initial steady state. Let w^* and r^* be the wage rate and the interest rate at the initial steady state respectively, where the immigration policy at the initial steady state is α^* for all t . Let s^* be the amount of savings of each individual and the number of old native at the initial steady state. Let a^* and b^* be the government savings (or debt if it is negative) divided by the number of young at the initial steady state and the social security benefit at the initial steady state. The government will choose the steady state lump-sum tax policy τ^* so that it satisfies the government budget constraint. This implies that at such b^* and τ^* , the resource constraint must be satisfied. Conversely, when the resource constraint is satisfied, then the government budget constraint is also satisfied.

The initial steady-state economy with the steady state immigration policy α^* is

⁶Using the homogeneity of the production function, we have $F(L_t, K_t) + (1 - \delta)K_t = F_k K_t + F_L L_t + (1 - \delta)K_t$. Then, using (2), $F(L_t, K_t) = (r_t + \delta)K_t + w_t L_t + (1 - \delta)K_t$. Using the definition of K_t and L_t and using the individual budget constraint, we have (8).

characterized as follows:

$$(s^*, l^*) = \arg \max_{s, l} u^y(w^*l - \tau^* - s, l) + v^y(g^y, g^{ind}) + \frac{1}{1 + \rho} [u^o((1 + r^*)s + b^*) + v^o(g^o, g^{ind})] \quad (9)$$

$$\text{where } w^* = \frac{\partial F(L^*, K^*)}{\partial L}, \quad r^* + \delta = \frac{\partial F(L^*, K^*)}{\partial K} \quad (10)$$

$$L^* = l^* R(\alpha^*) N_0^{n*} (1 + \alpha^*) \text{ and } K^* = (s^* + a^*) \times N_0^{n*} (1 + \alpha^*) \quad (11)$$

$$F(K^*, L^*) + (1 - \delta)K^* = \{c^{y*} + s^* + g^y + g^{ind} + a^*\} R(\alpha^*) N_0^{n*} (1 + \alpha^*) \quad (12)$$

$$+ \{c^{o*} + g^o + g^{ind}\} N_0^{n*} (1 + \alpha^*) \quad (13)$$

$$c^{y*} = w^* l^* - \tau^* - s^* \text{ and } c^{o*} = (1 + r^*) s^* + b^* \quad (14)$$

$$N_0^{n*} \text{ is some positive number} \quad (15)$$

The the utility level at the initial steady state is defined as follows:

$$u^* \equiv u^y(c^{y*}) + v^y(g^y, g^{ind}) + \frac{1}{1 + \rho} [u^o(c^{o*}) + v^o(g^o, g^{ind})] . \quad (16)$$

3.1.1 Welfare Effect of Increasing the INR

In this sub-subsection, I examine, starting from period 1, whether or not increasing the INR to a higher level permanently will Pareto-improve welfare. Increasing the INR permanently is defined as $\alpha_0 = \alpha^*$ and $\alpha_t = \alpha$ where $\alpha > \alpha^*$ for $t \geq 1$. For the analysis, consider the following constrained maximization problem, which is a function of α :

$$\begin{aligned}
V(\alpha) &= \max_{\{c_t^y, c_t^o, s_t, a_t | t=1,2,\dots\}} \frac{1}{1+\rho} [u^o(c_1^o) + v^o(g_o, g^{ind})] \\
\text{s.t. } &u^y(c_t^y, l_t) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho} [u^o(c_{t+1}^o) + v^o(g^o, g^{ind})] \geq u^* \text{ for } t = 1, 2, \dots \quad (17) \\
&F(L_t, K_t) + (1-\delta)K_t \geq \\
&\{c_t^y + s_t + g^y + g^{ind} + a_t\} N_{t-1}^n R(\alpha_{t-1})(1+\alpha_t) + \{c_t^o + g^o + g^{ind}\} N_{t-1}^n (1+\alpha_{t-1}) \text{ for } t = 1, 2, \dots \quad (18)
\end{aligned}$$

$$K_t = (s_{t-1} + a_{t-1}) N_{t-1}^n (1+\alpha_{t-1}) \text{ for } t = 2, \dots \text{ and } s_0 = s^* \text{ and } a_0 = a^*$$

$$L_t = l_t N_{t-1}^n R(\alpha_{t-1})(1+\alpha_t) \quad (19)$$

α and α^* are given

The above programming problem deserves several comments. First, $V(\alpha)$ is the utility of cohort 0 at period 1 when the government is accepting immigrants with a constant ratio α^* at the initial steady state and starts to accept immigrants with ratio α from period 1. Because at the period 1, the consumption of the cohort 0 at the young period is already determined, I do not include the consumption of cohort 0 at the young period. Second, the first constraint is related to Pareto improvement and requires that all cohorts except cohort 0 need to have at least as the same utility as they would have at the initial steady state. Note that in the first constraint, there are α and α^* . From the point of period 1, the immigration policy at the period 0 is pre-determined. Such a variable is denoted as α^* . The policy that is determined at the period 1 or later periods is denoted as α .

Note that N_{t-1}^n is determined by α and N_{t-2}^n , but N_{t-1}^n is also affected by α and N_{t-3}^n . This implies that a change of immigration policy α affects all N_t^n for $t = 1, 2, \dots$. To make the calculation easy, it is useful to divide the resource constraint by N_{t-1}^n

when $t = 1$ and by $N_{t-1}^n(1 + \alpha)$ when $t = 2, 3, 4, \dots$ ⁷ Then, (18) becomes as follows:

$$F(R(\alpha^*)(1 + \alpha)l_1, (s^* + a^*)(1 + \alpha^*)) + (1 - \delta)(s^* + a^*)(1 + \alpha^*) \geq \{c_1^y + s_1 + g^y + g^{ind} + a_1\}R(\alpha^*)(1 + \alpha) + \{c_1^o + g^o + g^{ind}\}(1 + \alpha^*) \text{ for } t = 1 \quad (20)$$

$$F(R(\alpha)l_t, s_{t-1} + a_{t-1}) + (1 - \delta)(s_{t-1} + a_{t-1}) \geq \{c_t^y + s_t + g^y + g^{ind} + a_t\} \times R(\alpha) + \{c_t^o + g^o + g^{ind}\} \text{ for } t = 2, 3, 4, \dots \quad (21)$$

Let L be the Lagrangian function. Let γ_t and λ_t be the Lagrangian multiplier of the minimum utility constraint (17) and the resource constraints (20) and (21). Let γ_t^* and λ_t^* be the Lagrangian multipliers when $\alpha = \alpha^*$. Then, we have the following observation.

Observation 1

When $\alpha = \alpha^$ the solution of MPP is*

$$c_t^y = c^{y*}, \quad c_0^o = c^{o*}, \quad s_t = s^*, \quad a_t = a^*, \quad l_t = l^* \text{ for } t = 1, 2, \dots \quad (22)$$

$$\lambda_1^* = \frac{1}{1 + \rho} u_c^o(c_1^o) \text{ and } \lambda_{t+1}^* = \frac{R(\alpha^*)}{1 + r^*} \lambda_t^* \quad (23)$$

$$\gamma_t^* = \frac{1}{u_c^y(c^{y*}, l^*)} \lambda_t^* R(\alpha^*) \text{ and for } t = 1, 2, \dots \quad (24)$$

For the proof of observation 1, see appendix B1.

Observation 1 implies that when the immigrant-native ratio α is fixed at α^* , the initial steady state allocation is Pareto-efficient and it is not possible to have Pareto improvement from the initial steady state holding $\alpha = \alpha^*$. Now suppose that the government increases the INR from α^* . Whether or not such an increase of the INR Pareto-improves welfare can be analyzed by calculating $dV/d\alpha$ and evaluating it at $\alpha = \alpha^*$. From the envelope theorem, $dV/d\alpha|_{\alpha=\alpha^*}$ is equal to

⁷The reason that we did divide the resource constraint at the period 1 by N_{t-1}^n , not $N_{t-1}^n(1 + \alpha)$ is that the population of the old at the period 1 is $N_{t-1}^n(1 + \alpha^*)$, not $N_{t-1}^n(1 + \alpha)$.

$$\left\{ R(\alpha^*)\lambda_1^* + \sum_{t=2}^{\infty} \lambda_t^* R'(\alpha^*) \right\} \times \{ F_L(R(\alpha^*), s^* + a^*)l^* - (c^{y*} + s^* + g^y + g^{ind} + a^*) \}. \quad (25)$$

$$\text{where } R(\alpha) = 1 + \pi_n + \alpha(1 + \pi_m) \quad (26)$$

The first bracket is positive because the Lagrangian multiplier of the resource constraint is positive and the marginal effect of increasing α on one plus the population growth rate is positive. In the second bracket, the first term is the marginal product of labor times labor supply, which is what an individual contributes to the economy when he or she is young at the initial steady state. When an young individual contribute $F_L l^*$ to the economy, there are three choices to distribute this contribution from the point of the government. The first choice is to let the young individual consume this contribution. The second choice is to transfer this contribution to the future periods and let this young individual or future cohort to consume it. The third choice is to transfer this contribution to the old individuals. $c^{y*} + g^y + g^{ind}$ is the amount of the resource consumed by the current young individual at the initial steady state. $s^* + a^*$ is the amount of the resource that is transferred to future periods. Note that $c^{y*} + g^y + g^{ind} + s^* + a^*$ does not include c^{o*} and g^o . Thus, $\{F_L l^* - c^{y*} - g^y - g^{ind} - s^* - a^*\}$ is the amount of the resource transferred to the old individuals at the initial steady state. We call this amount as the upward intergenerational transfer.

Definition: When $F_L l^* - c^{y*} - g^y - g^{ind} - s^* - a^*$ is positive, we say that the MPL condition is satisfied and call the amount $F_L l^* - c^{y*} - g^y - g^{ind} - s^* - a^*$ as upward intergenerational transfers.

Proposition 1 (MPL condition version) *If there exit upward intergenerational transfers, in the sense that the marginal product of labor of a young individual times labor supply is greater than the sum of resources that a young individual consumes when he or she is young and the amount of resources that are transferred to future periods, then accepting more immigrants Pareto-improves the welfare of all genera-*

tions.⁸ Because the marginal product of labor of the young times labor supply is the pre-tax earning of a young individual and because $c^{y*} + s^*$ is the after-tax income of the young by the definition, $F_L l^* - c^{y*} - s^*$ is the amount of tax paid by a young individual. $g^y + g^{ind} + a^*$ is the amount of resources provided by the government to a current young individual and the amount of resources to the future cohorts per each young. Thus, we have the following Corollary:

Corollary (Tax-expenditure version) *Alternatively, if the amount of tax paid by the young is greater than the sum of the amount of resources provided by the government to a current young individual and the amount of resources to the future cohorts per each young at the initial steady state, accepting more immigrants Pareto-improves welfare.*

This MPL condition plays a critical role for the analysis not only of the case where the government has an access to the lump-sum tax but also of the case where the government does not have an access to the lump-sum tax. In addition, in the simulation analysis, this MPL condition plays an important role.

The Tax-expenditure condition has also an important implication for the cost-benefit analysis of accepting immigrants. Note that for the government expenditure part, the social security benefit and publicly provided private goods for the old are not included in the Tax-expenditure condition. Only the tax that young immigrant pays and the resource that is used for the young or the future cohorts should be included.

⁹¹⁰ Graphically, Proposition 1 is explained in Appendix A1.

⁸Note that this condition does not change even in the presence of public goods because an increase of the number of immigrants does not affect the consumption of public goods by the nature of public goods.

⁹When immigrants and natives are different in preferences and productivities, then the sum of the redistribution from immigrants to natives and MPL condition become important. See section 3.3.

¹⁰In a typical study of assessing the fiscal effect of accepting more immigrants, it calculates the present value of the government expenditure such as the consumption of publicly provided private goods and the social security benefit that immigrants receive and the tax revenue (including income tax and the social security tax) that immigrants pay. However, if the social security benefit that the retired immigrants receive is included in the cost-benefit calculation, then the social security tax that the children of immigrants pay should also be included because the social security benefits are financed by the social security tax that the children of natives and immigrants pay. Of course, if the social security tax that the children of immigrants pay is included, then the social security benefit that the children of the immigrants receive should also be included. Again, if the social security

3.2 Presence of Distorting Taxes and Implementation of Pareto Improvement

In the preceding subsection, it was shown that it is Pareto-improving to accept more immigrants when the lump-sum transfers are possible and when there are upward intergenerational transfers from the young to the old in the sense that the marginal product of labor of the young times labor supply is greater than the resources that are allocated to the young or to the future cohort. With a neoclassical production function that exhibits diminishing marginal product, the pre-tax wage decreases and the pre-tax interest rate increases when more immigrants are accepted. In the absence of the lump-sum transfer, it is not clear whether it is possible to Pareto-improve all generations.

In this subsection, I will show that, it is possible to Pareto-improve welfare, without changing incentives of individuals, by increasing INR in the absence of the lump-sum transfer as long as the MPL condition is satisfied¹¹ In addition, I show that a relatively simple adjustment of taxes(wage tax and interest tax) and social security benefit achieves this Pareto improvement while the government is increasing the INR.

For the analysis, let $\tilde{\alpha}$ be a time invariant new immigration policy from period 1 where $\tilde{\alpha} > \alpha^*$. As in the previous section, I assume that the economy is dynamically efficient at the initial steady state. This implies that

$$F_K(R(\alpha^*), s^* + a^*) > \delta + R(\alpha^*) - 1 \quad (27)$$

In this sub-section, we analyze whether government can make the economy reach the golden rule by accepting more immigrants in a Pareto-improving way. To avoid a

benefit of the children of the immigrants receives is included, then the social security tax that the grand-children of the immigrants should be included. Note that in the PYGO social security system, the social security benefit that the immigrants receives is roughly balanced by the social security tax paid by the children of the immigrants. This implies that in the cost-benefit calculation, the social security benefit that the immigrant receive is roughly canceled out by the social security tax that the children of the immigrants pay. Thus, in the cost benefit calculation, only the tax that young immigrants pay and the publicly provided private goods for the young immigrants should be included.

¹¹In the presence of non-optimal taxation, it is obvious that it is possible to Pareto-improve welfare if the government can change incentives of individuals.

situation that the golden rule level capital stock does not exist, I assume that the sum of the population growth rate and the depreciation rate is strictly greater than zero:

$$R(\alpha_{t-1}) - 1 + \delta > 0 \text{ for } \alpha_{t-1} \geq 0. \quad (28)$$

As for taxes, I assume that the government uses a capital income tax and a wage tax at the initial steady state. I assume that those taxes do not need to be the second best optimal. Let τ_{wt} and τ_{rt} be the wage tax rate and capital income tax rate at the period t . Let the individual budget constraint (7) is modified as follows:

$$w_t l_t (1 - \tau_{wt}) = c_t^y + s_t \text{ and } b_{t+1} + (1 + (1 - \tau_{rt+1})r_{t+1})s_t = c_{t+1}^o$$

Let τ_w^* and τ_r^* be the wage tax rate and capital income tax rate at the initial steady state. At the initial steady state, the above budget constraints become as follows:

$$w^* l^* (1 - \tau_w^*) = c^{y^*} + s^* \text{ and } b^* + (1 + (1 - \tau_r^*)r^*)s^* = c^{o^*}$$

I assume that at the initial steady state the social security benefit is proportional to pre-tax earnings:

$$b^* = \Omega \times w^* l^* \quad (29)$$

When the government increases the INR, the wage rate falls and the interest rate increases initially. To achieve Pareto improvement by increasing the INR, first I assume that the government sets the tax rates such that after-tax wage and interest rate after an increase of the INR are equal to the after-tax wage rate and interest rate at the initial steady state. Then, the wage tax rate and the interest tax rate for period t are set as follows:

$$w_t (1 - \tau_{wt}) = w^* (1 - \tau_w^*) \text{ and } r_t (1 - \tau_{rt}) = r^* (1 - \tau_r^*) \quad (30)$$

Second, I assume that the government re-scales Ω so that the social security benefit becomes proportional to after-tax earnings, not pre-tax earnings, and that an indi-

vidual receives the same benefit when wage rate, wage tax rate and labor supply are at the same level as at the initial steady state. This implies that

$$b_{t+1} = \frac{\Omega}{1 - \tau_w^*} w_t (1 - \tau_{wt}) l_t \quad (31)$$

Note that $b_{t+1} = \Omega w^* l_t$ when $w_t (1 - \tau_{wt}) = w^* (1 - \tau_w^*)$.

When the government sets taxes and social security benefit in this way, saving behavior and labor supply behavior do not change because the budget constraint of a consumer at any period t is the same as at the initial steady state. This implies that the equilibrium social security benefit at any period t is the same as at the initial steady state. If the government provides at least as the same level of publicly provided private goods, the levels of the utility of all cohorts are at least as the same as at the initial steady state.

As for the extent of the increased immigrants, motivated by Proposition 1, I assume that the MPL condition is satisfied at the initial steady state:

$$l^* F_L(l^* R(\alpha^*), s^* + a^*) > c^{y^*} + g^y + g^{ind} + s^* + a^* \quad (32)$$

The LHS of the above equation is a marginal increase of the output due to a one unit increase of the population. The RHS is the amount of resources that an individual receives when he or she is young.

The result that was generated in the preceding subsection shows that a marginal increase in the number of immigrants Pareto-improves welfare if the MPL condition is satisfied. However, that result does not imply that an unlimited acceptance of immigrants always Pareto-improves welfare. I impose two conditions on the extent that immigrants are accepted. The first condition is regarding the MPL condition when the new immigration policy $\tilde{\alpha}$ is implemented. I assume that when the new immigration policy $\tilde{\alpha}$ is operative and when the government savings per each young is held constant, the marginal increase of the output due to a one unit increase of the population is greater or equal to the amount of resources that an individual receives when he or she is young at the initial steady state. This implies that

$$l^* F_L(l^*(1+\tilde{\alpha})((1+\pi_n)N_{t-1}^n + (1+\pi_m)N_{t-1}^m), (s^*+a^*)(N_{t-1}^n + N_{t-1}^m)) \geq c^{y*} + g^y + g^{ind} + s^* + a^* . \quad (33)$$

Note that $(1+\pi_m) \times N_{t-1}^m + (1+\pi_n)N_{t-1}^n$ is the population of young native at period t . Using the homogeneity of the production function and $N_{t-1}^m = \tilde{\alpha}N_{t-1}^n$, (33) can be written as

$$l^* F_L(l^*(R(\tilde{\alpha}), s^* + a^*) \geq c^{y*} + s^* + g^y + g^{ind} + a^* . \quad (34)$$

The second condition is regarding the golden rule. When the government accepts more immigrants according to (34), and it adjusts the wage tax rate and capital income tax rate so that the after-tax wage rate and after-tax interest rate are the same as at the initials steady state as defined in (30), it is possible that the golden rule binds even the government savings per each young is held constant. This is more likely when the capital stock per capita at the initial steady state is lower than the golden rule level very close to the golden rule level. This implies that at $\tilde{\alpha}$, the following golden rule is satisfied.

$$F_K(R(\tilde{\alpha}), s^* + a^*) = \delta + R(\tilde{\alpha}) - 1 \quad (35)$$

On the other hand, when the capital stock per capita at the initial steady state is sufficiently lower than the golden rule level, the MPL condition (34) binds first instead of the golden rule (35) when the government accepts more immigrants. This implies that the following condition holds at $\tilde{\alpha}$ when the government savings per each young is held constant:

$$F_K(R(\tilde{\alpha}), s^* + a^*) > \delta + R(\tilde{\alpha}) - 1 \quad (36)$$

If (35) holds, then the government does not need to increase the government savings to reach the golden rule and the analysis becomes trivial. Thus, in the following analysis I assume that (36) holds. In other words, I assume that the capital stock per capita at the initial steady state is sufficiently lower than the golden rule level.

When the government accepts more immigrants, the government can increases the

government savings balance as I show below. This implies that it is possible that, at some point, the marginal product of capital (MPK) becomes equal to the golden level of capital stock per capita. But when the MPK becomes equal to the golden rule level, it is clearly better to use the entire government surplus to increase the supply of publicly provided private goods rather than to increase the government savings balance. Thus, I assume that as long as the MPK is higher than the golden rule level, the government uses some of the government budget surplus to increase the government saving balance and the remainder to increase the supply of publicly provided private goods. When the MPK reaches the golden rule level, the government uses all of the surplus to increase the publicly provided private goods. Thus, we have the following MPK condition:

$$F_K(R(\tilde{\alpha}), s^* + a_t) \geq \delta + R(\tilde{\alpha}) - 1 \quad (37)$$

$$\text{where } a_t > a^* \quad (38)$$

Now we examine whether social security benefit and taxes determined by (31) and (30) are feasible from the point of the government budget constraint. To check the feasibility of such taxes, consider the net government budget surplus for period 1, SP_1 .

$$SP_1 = (w_1 \tau_{w1} l^* - g^y - g^{ind} - a^*) \sum_{j=n,m} N_1^j + (r_1 \tau_{r1} s^* - b^* - g^o - g^{ind}) \sum_{j=n,m} N_0^j + (1 + r_1) a^* \sum_{j=n,m} N_0^j \quad (39)$$

Note that τ_{w1} and τ_{r1} are defined in (30). Also I assume that the government will have at least the same amount of the government saving per the number of the young as at the initial steady state. By substituting τ_{w1} and τ_{r1} into SP_1 and using the homogeneity of the production function, we have (see appendix B2)

$$SP_1 = N_1^n \int_{1+\alpha^*}^{1+\tilde{\alpha}} F_L(N_1^n l^* z, (s^* + a^*) N_0^n (1 + \alpha^*)) l^* - c^{y*} - s^* - g^y - g^{ind} - a^*] dz \quad (40)$$

where $N_1^n = N_0^n (1 + \pi_n) + N_0^n (1 + \alpha^*) (1 + \pi_m)$. Thus, from the MPL conditions

(32) and (33), the inside of the integration is positive for $z \in [1 + \alpha^*, 1 + \tilde{\alpha}]$. This means that this tax plan is feasible in period 1. The government can use some of the surplus of the budget to increase the supply of publicly provided private goods and put remainder in savings. Let a_1 be the balance of the government savings per each young individual at the end of period 1 where $a_1 > a^*$. What will be the net budget surplus in period 2, SP_2 ?

Note that the natives and immigrants of cohort 1 will save the same as the natives and immigrants at the initial steady state, because they face the same after-tax wage rate and interest rate under the proposed tax policy as at the initial steady state. This implies that $s_1 = s^*$ for both groups of cohort 1. Assume that at the period 2, the government will save at least a^* per the number of the young. Thus, SP_2 becomes

$$SP_2 = \sum_{j=n,m} N_2^j \times (w_2 \tau_{w2} - g^y - g^{ind} - a^*) + \sum_{j=n,m} N_1^j \times (r_2 \tau_{r2} s^* - b^* - g^o - g^{ind}) + (1 + r_2) a_1 \sum_{j=n,m} N_1^j. \quad (41)$$

Note that the pre-tax wage for period 2, w_2 , and the pre-tax interest rate for period 2, r_2 , are equal to

$$w_2 \equiv F_L(l^*(N_2^n + N_2^m), (N_1^n + N_1^m)(s^* + a_1)) \text{ and } r_2 \equiv F_K(l^*(N_2^n + N_2^m), (N_1^n + N_1^m)(s^* + a_1)) - \delta \quad (42)$$

Again the government sets τ_{w2} and τ_{r2} such that after-tax wage rate and after-tax interest rate become the same as at the initial steady state. Thus, SP_2 becomes (see appendix B3)

$$\begin{aligned} SP_2 = & N_1^n (1 + \tilde{\alpha}) \int_{s^* + a^*}^{s^* + a_1} [F_K(l^* N_2^n (1 + \tilde{\alpha}), z N_1^n (1 + \tilde{\alpha})) + 1 - \delta] dz \\ & + (1 + \tilde{\alpha}) N_1^n \int_{\alpha^*}^{\tilde{\alpha}} R'(\alpha) [F_L(l^* R(z), (s^* + a^*)) l^* \\ & - \{c^{y*} + s^* + g^y + g^{ind} + a^*\}] dz \end{aligned} \quad (43)$$

The first term of (43) measures the welfare gain that arises from the additional savings that the government accumulates at the end of period 1. The second term measures

the welfare gain that arises from the increased population growth rate in the presence of the PYGO social security. From the MPK condition (??) and the condition on the population growth rate the inside of the first integration is positive. From (34), the inside of the second integration is positive. Thus, SP_2 is positive and the government can implement the proposed tax policy. Again, at the end of period 2, the government can use some of the above surplus to increase the supply of publicly provided private goods and put the rest to increase the balance of government savings. Similarly, the government surplus for period t becomes

$$\begin{aligned}
SP_t = & N_{t-1}^n (1 + \tilde{\alpha}) \int_{s^* + a^*}^{s^* + a_{t-1}} [F_K(l^* R(\tilde{\alpha}), z) + 1 - \delta] dz \\
& + (1 + \tilde{\alpha}) N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} R'(\alpha) [F_L(l^* R(z), (s^* + a^*)) l^* \\
& - \{c^{y^*} + s^* + g^y + g^{ind} + a^*\}] dz
\end{aligned} \tag{44}$$

where a_t is the balance of government saving per each young individual at the end of period $t - 1$. Again the government uses some of the surplus for increasing publicly supplied private goods and puts the rest in savings. This implies that $SP_t > 0$ for all $t = 1, 2, \dots$. Thus, we have the following proposition:

Proposition 2. *Consider an economy in which the wage and interest taxes are used at the initial steady state. If the MPL condition is satisfied at the initial steady state, accepting more immigrants with tax rule (30) and social security benefit rule (31) Pareto-improves the welfare of all generations.*

3.2.1 Government Savings and the Golden Rule

In this subsection, I examine the capital stock path and government saving path when the government increases the INR. To examine the government saving path, I need to specify how much of the government surplus, SP_t , is put into the additional government savings. For the analysis, I assume that the surplus that arises from the increased government savings at period $t - 1$, which is the first integration of SP_t ,

is put into the additional government savings that is added to a^* at period t . Note that the government could use some part of the second integration in SP_t , the surplus that is generated directly from the increased immigration. Thus, my assumption is a conservative value for the government savings. I will show that, even with this conservative level, the economy reaches the golden rule level of capital stock per capita within a finite time in a Pareto-improving way. Note that the total number of young individuals at the period t is $N_{t-1}^n R(\tilde{\alpha})(1 + \tilde{\alpha})$. Thus, the government savings per each young individual at the end of period t for $t \geq 2$, a_t , becomes Q

$$a_t - a^* = \frac{N_{t-1}^n(1 + \tilde{\alpha})}{N_{t-1}^n R(\tilde{\alpha})(1 + \tilde{\alpha})} \int_{s^* + a^*}^{s^* + a_{t-1}} [F_K(l^* R(\tilde{\alpha}), z) + 1 - \delta] dz \quad (45)$$

$$\equiv Q(a_{t-1}) \quad (46)$$

Note that from (37), we have

$$\begin{aligned} a_t - a^* &\geq \frac{N_{t-1}^n(1 + \tilde{\alpha})}{N_{t-1}^n R(\tilde{\alpha})(1 + \tilde{\alpha})} \int_{s^* + a^*}^{s^* + a_{t-1}} R(\tilde{\alpha}) dz \\ &= a_{t-1} - a^* \end{aligned}$$

Thus, a_t is increasing over time as long as a_t is determined according to equation (45) and (37). Now, consider the graph of $a_t = Q(a_{t-1}) + a^*$ where a_t is measured on the vertical axis and a_{t-1} is measured on the horizontal axis. $Q(a_{t-1}) + a^*$ is equal to a^* at $a_{t-1} = a^*$ and $Q(a_{t-1}) + a^*$ is increasing. It is also concave due to the diminishing marginal product of capital. The slope of $Q(a_{t-1}) + a^*$ at $a_{t-1} = a^*$ is

$$Q'(a^*) = \frac{1}{R(\tilde{\alpha})} (F_K(l^* R(\tilde{\alpha}), s^* + a^*) + 1 - \delta) \quad (47)$$

Because of assumption (36), $Q'(a_{t-1})$ at $a_{t-1} = a^*$ is strictly greater than 1. On the other hand, due to the diminishing marginal product of capital, $Q'(a_{t-1})$ becomes close to zero as a_{t-1} becomes bigger. Thus, the $a_t = Q(a_{t-1}) + a^*$ and 45 degree line intersect at $a_{t-1} = a^*$ and at a^{***} where $a^{***} > a^*$. Let a^{**} be the point where $Q'(a^{**})$

= 1. This implies that at a^{**}

$$\frac{\partial F(l^* R(\tilde{\alpha}), s^* + a^{**})}{\partial K} + 1 - \delta = R(\tilde{\alpha})$$

In other words, at a^{**} the golden rule is satisfied. Note that the government can choose a_1 so that $a_1 > a^*$ because the surplus at period 1 is strictly positive. From the graph of $a_t = Q(a_{t-1}) + a^*$, a_t keeps increasing starting from a small $a_1 > a^*$. Before it reaches a^{***} , it reaches a^{**} within a finite time. This implies that the economy reaches the golden rule level of capital stock per capita within a finite time.

Note first that in this analysis, I assume that only the first integration of SP_t , i.e. the surplus that arises from the increased government savings at period $t - 1$, is put into savings at period t . However, the second integration of SP_t , i.e. the surplus which arises directly from increased immigrants, also can be put into savings. Thus, the government can shorten the time to reach the golden rule level by putting the surplus that arises directly from increased immigrants. Secondly, we can note that the government can induce the economy to reach the modified golden rule level within a finite time in a Pareto-improving way, because the modified golden rule level is lower than the golden rule level.

Proposition 3. (Reaching (modified) Golden Rule Level) *Suppose that a PYGO social security system is used initially and that the MPL condition is satisfied. Also assume that the capital stock per capita at the initial steady state is sufficiently lower than the golden rule level. Then, by accepting more immigrants and using the proposed tax rule, social security benefit rule and government saving policy, the government can induce the economy to reach the (modified) golden rule level within a finite time in a Pareto-improving way.*

3.3 Intra-redistributional Channel and Difference of Productivities and Preferences

When immigrants earn less than natives or immigrants consume more publicly provided private goods than natives, accepting more immigrants could decrease the wel-

fare of natives, because it means that more resources are taken from natives and used for immigrants. This is the intra-redistributional channel of accepting more immigrants. A similar redistribution could also happen when the preferences of immigrants and natives are different and the labor supply or savings of immigrants differs from those of natives. To analyze this redistributional channel of accepting more immigrants, consider again a permanent change of immigration policy such that $\tilde{\alpha} > \alpha^*$. Let $(c^{y,j*}, c^{o,j*}, s^{j*}, l^{j*})$, be the consumption at the young period, the consumption at the old period, savings and labor supply of type j ($j = n, m$), respectively at the initial steady state. Since I assume that the preferences and the productivities of the native and immigrant are different in this section, I put superscript j even for the variables at the steady-state situation. Also, let $g^{i,j}$ be the publicly provided private goods for goods type i and type j nationality where $i = y, o, ind$ and $j = n, m$. For conditions that guarantee that accepting more immigrants Pareto-improves welfare, I assume that the following condition is satisfied:

$$\begin{aligned}
\text{Modified MPL condition} &= R'(\alpha) \left[\frac{\partial F}{\partial L} l^{n*} - (c^{y,n*} + g^{y,n} + g^{ind,n} + s^{n*} + a^*) \right] \\
&+ J'(\alpha) \left[\frac{\partial F}{\partial L} \phi^m l^{m*} - (c^{y,m*} + g^{y,m} + g^{ind,m} + s^{m*} + a^*) \right] \\
&+ \left(\frac{\partial F}{\partial K} + (1 - \delta) \right) (s^{m*} + a^*) - (c^{o,m*} + g^{o,m} + g^{ind,m}) > 0 \\
&\text{for all } \alpha \in [\alpha^*, \tilde{\alpha}] : \\
\text{where } \frac{\partial F}{\partial L} &= \frac{\partial F(R(\alpha)l^n + \phi^m J(\alpha)l^m, s^{n*} N_{t-1}^n + \alpha s^{m*})}{\partial L}, \\
\frac{\partial F}{\partial K} &= \frac{\partial F(R(\alpha)l^n + \phi^m J(\alpha)l^m, s^{n*} + \alpha s^{m*})}{\partial K} \\
&\text{and } J(\alpha) = (1 + \pi_n + \alpha(1 + \pi_m))\alpha
\end{aligned} \tag{48}$$

In the first line and the second line, the inside of the bracket is the amount of the upward intergenerational transfer made by a native and an immigrant, respectively. Thus, the sum of the first line and the second line is essentially the same as the MPL condition in the preceding section. The third line captures the intra-redistribution from the natives to immigrants. The third line measures the amount that an old

immigrant contribute to the economy though savings minus the resource used for an old immigrant. When the productivities and preferences of immigrants and natives are the same, the third line becomes equal to the inside of the first line, the amount of the upward intergenerational transfer. Thus, the above modified MPL condition states that the total upward intergenerational transfer is large enough to offset the intra-redistribution from an old immigrant to an old native. From (48), we have the following propositions.¹²

Proposition 4 *Assume that natives and immigrants do not have same productivities and preferences. If the Modified MPL condition is satisfied, it is possible to Pareto-improve the welfare of all generations by accepting more immigrants. For the proof, see Appendix B4.*

Proposition 5 *If the Modified MPL condition is satisfied, the government can induce the economy to reach the (modified) golden rule level within a finite time in a Pareto-improving way by accepting more immigrants.*

4 Quantifying the Welfare Gain of Accepting More Immigrants in the Presence of a PYGO Social Security System

Propositions 1 and Proposition 2 above show that accepting more immigrants can Pareto-improve the welfare of all generations if there are intergenerational transfers in the sense that the marginal product of labor of a young individual is greater than what he or she receives including publicly provided private goods while he or she is young (the MPL condition). Furthermore, Proposition 3 shows that if the government can put some of the welfare gain into savings, it is possible to induce the economy to reach the golden rule level of capital in a Pareto-improving way within a finite time. This result is in sharp contrast to findings in the literature on social security reform which shows that some generation must bear a double burden to increase the capital

¹²In Appendix B5, I show that the modified MPL condition becomes the MPL condition when immigrants and natives have the same productivities and preferences.

stock of the economy in the presence of a PYGO social security system (Geanakoplos, Mitchell and Zeldes 1998). In addition, the proposition 4 and 5 show that even if an immigrant earns less than a native, accepting more immigrants Pareto-improves welfare when the intra-redistribution is not as large as the intergenerational transfer from the young to the old.

A number of issues arise as a result of accepting proposition 1-5. First, those propositions are based on a two-period overlapping generation model. In a realistic multi-period overlapping generation model, it might not be possible for an economy to reach the (modified) golden rule level of capital stock per capita within a finite time in a Pareto-improving way by accepting more immigrants. Second, although proposition 3 shows that the economy reaches the (modified) golden rule level of capital stock per capita within a finite time, in practice it might take a long time, as long as 1000 years, to reach the golden rule level. Third, propositions 1-5 are silent on the quantitative effect on welfare. Given that it might take a quite long time to reach the golden rule level, the welfare gain of accepting more immigrants can be very small. In addition, propositions 4 and 5 say nothing about the degree to which the difference in productivity between immigrants and natives is allowed for with respect to Pareto improvement when more immigrants are accepted.

This section addresses these issues. To address these issues, I use the computational overlapping generation model developed by Auerbach and Kotlikoff (1987). I assume that the model economy consists of overlapping generations in which each generation lives for 80 periods and that the probability of death increases with each passing period. I assume that the model economy is similar to the US economy in several dimensions.¹³ In the analysis, I assume that initially the model economy is on the balanced growth path and the percentage of immigrants to natives (PITN), which is defined as 100 times the number of immigrants divided by the number of natives, is similar to that which the US 2000 census data indicates. Then, I will ex-

¹³However, the model economy is different in several important dimensions as well. For example, the model economy does not include a certain aspect of open economy such as international trade and capital mobility. The model economy does not incorporate the accumulation of human capital of the natives and immigrants.

amine whether it is possible to Pareto-improve the welfare of all the generations in the model economy by increasing the PITN by a reasonable size. In addition, I examine how long it takes for the model economy to reach the (modified) golden-rule level in a Pareto-improving way and quantify the Pareto-improving welfare gain. To check the robustness, I recalculate the model by changing the value of the following parameters: the replacement, initial government debt (asset) level, the level of earnings of immigrants, immigrants' consumption of publicly provided private goods, probability of immigrants returning to the country of origin, the CRRA and the time preference rate.

4.1 The Model Economy: Auerbach and Kotlikoff Model with Immigration

Agents will show up in the model from age 1. I assume that age 1 corresponds to age 20 in real life. They work from age 1 until age 45. From the beginning of age 46, they retire. At each age, they die with some probability and they can live until age 80. Let i be the index of age. For $i \geq 2$, let p_i be the probability that an agent is alive at the age i , given that he or she is alive until age $i - 1$. Due to the lack of data, I assume that p_i is the same for natives and immigrants. To simplify the notation, I assume that $p_1 = 1$.¹⁴ An agent who enters the model at period t maximizes the following utility function:

$$\max \sum_{i=1}^{45} \beta^i \prod_{q=1}^i p_q \left\{ \frac{[(c_{t-1+i}^{i,j})^\zeta (1 - l_{t-1+i}^{i,j})^{1-\zeta}]^{1-\gamma}}{1 - \gamma} + g_{t-1+i}^{ij} \right\} + \sum_{i=46}^{80} \beta^i \prod_{q=1}^i p_q \left\{ \frac{[c_{t-1+i}^{i,j}]^{\zeta(1-\gamma)}}{1 - \gamma} + g_{t+1-i}^{ij} \right\} \quad (49)$$

where $c_{t-1+i}^{i,j}$ and $l_{t-1+i}^{i,j}$ are the amounts of private consumption and labor supply of type j agent of age i at period $t - 1 + i$. g_{t+1-i}^{ij} is the amount of publicly provided private goods to age i agent in period $t + 1 - i$. I assume that the amount of g_{t+1-i}^{ij} is chosen by the government. In this formulation, as in Storesletten (2000), I postulate

¹⁴Infant and child mortality is defined in (55).

that the immigrants assume they will stay in the host country until the end of their lives.¹⁵ Let $s_t^{i,j}$ be the savings of type j of age i at time t . The budget constraint of an agent at age i is

$$s_{t-2+i}^{i-1,j}(1 + r_{t-1+i}(1 - \tau_{r,t-1+i})) + (1 - \tau_{w,t-1+i})w_{t-1+i}H^{i,j} \times l_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 1 \leq i \leq 45 \quad (50)$$

$$s_{t-2+i}^{i-1,j}(1 + r_{t-1+i}(1 - \tau_{r,t-1+i})) + b_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 46 \leq i \leq 80 \quad (51)$$

$$s_{t-1+i}^{i,j} = 0 \text{ for } i = 0 \text{ and } s_{t-1+i}^{i,j} \geq 0 \text{ for } 1 \leq i \leq 80 \quad (52)$$

where $H^{i,j}$ is the efficient unit of human capital of type j at age i and $H^{i,j} > 0$ for $1 \leq i \leq 45$ and $H^{i,j} = 0$ for $i \geq 46$. w_t is the wage rate for one efficient unit of labor at period t . I assume that an individual cannot have a negative savings balance. Once an individual dies, the government imposes a 100 percent inheritance tax. $b_{t-1+i}^{i,j}$ is the social security benefit for type j of age i that is given at period $t - 1 + i$. For $i \leq 45$, $b_{t-1+i}^{i,j} = 0$ and for $i \geq 46$, $b_{t-1+i}^{i,j}$ is determined as follows:

$$b_{t-1+i}^{i,j} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45} (1 + \mu)^{45-i} w_{t-1+i} l_{t-1+i}^{i,j} H^{i,j}}{45 \times 12}$$

where RR is the replacement and $AIME^j(t)$ is the average income monthly index of the cohort who become age 1 at time t .

For the production function, I assume that the economy's aggregate production can be described by the Cobb-Douglas production function:

$$Y_t = K_t^\theta (E_t L_t)^{1-\theta} \text{ and } \mu = (E_{t+1} - E_t)/E_t \quad (53)$$

where θ is the capital share and E_t represents the level of technology. μ is income per capita growth rate. L_t is the efficient unit of labor supply at period t .

Let $1 - \hat{p}_i$ be the probability that an immigrant returns to her/his home country

¹⁵Alternatively, we can assume that immigrants will enjoy the same wage level in their home country as in the host country even if they return to their home countries. The two assumptions generate the same results.

at the beginning of age i , given that she or he stays in the host country at the age $i - 1$ for $i \geq 2$. To simplify the notation, I assume that $\widehat{p}_1 = 1$. Let N_t^{ij} be the number of agent of age i of type j at time t . Then, $N_t^{im} = p_i \widehat{p}_i N_t^{i-1,m}$ and $N_t^{in} = p_i N_t^{i-1,n}$. L_t is defined as follows:

$$L_t = \sum_{i=1}^{45} \sum_{j=n,m} H^{i,j} N_t^{i,j} l_t^{i,j} \quad (54)$$

I assume that all immigrants will arrive in the host country at age 1. Let σ_n be the growth rate of the number of natives of age 1 at the steady state and let σ_m be the growth rate of the number of immigrants of age 1 at the steady state. The total number of natives is $\sum_{i=1}^{80} N_t^{in} = N_t^{1,n} \sum_{i=1}^{80} (1 + \sigma_n)^{-(i-1)} \times \prod_{q=1}^i p_q$. The total number of immigrants is $N_t^{1,m} \sum_{i=1}^{80} (1 + \sigma_m)^{-(i-1)} \times \prod_{q=1}^i p_q \widehat{p}_q$. This implies that the ratio of the total number of immigrants to the total number of natives is constant if and only if the growth rates of $N_t^{1,n}$ and $N_t^{1,m}$ are the same. Thus, for calculating the steady state, I assume that $\sigma_n = \sigma_m = \sigma$. I assume that the children of immigrants become native if their parents stay in the host country until the children attain adulthood. Then, $N_t^{1,n}$ is determined by the fertility rates of both immigrants and natives and the return rate of immigrants. Let $\eta_j^{i,j}$ be the age-specific fertility of type j at age i . The number of children born of type j parents of age i is $N_t^{i,j} \eta_j^{i,j}$. Let d be the infant-child mortality. Then the number of natives of age 1 at $t + 20$, $N_{t+20}^{1,n}$, is determined as follows:

$$N_{t+20}^{1,n} = (1 - d) \times \left\{ \sum_{i=1}^{80} \eta_j^{i,m} N_t^{i,m} \prod_{x=1}^{20} \widehat{p}_{i+x} + \sum_{i=1}^{80} \eta_j^{i,n} N_t^{i,n} \right\} \quad (55)$$

The percentage of immigrants to natives (PITN) at the steady state (see Appendix A6) becomes

$$\begin{aligned} \frac{\sum_{i=1}^{80} N_t^{i,m}}{\sum_{i=1}^{80} N_t^{i,n}} \times 100 &= \frac{\frac{1}{1-d} - \sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta_j^{i,n} \times \prod_{q=1}^i p_q}{\sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta_j^{i,m} \times \prod_{q=1}^i p_q \widehat{p}_q \prod_{x=1}^{20} \widehat{p}_{i+x}} \\ &\times \frac{\sum_{i=1}^{80} (1 + \sigma)^{-(i-1)} \times \prod_{q=1}^i p_q \widehat{p}_q}{\sum_{i=1}^{80} (1 + \sigma)^{-(i-1)} \times \prod_{q=1}^i p_q} \times 100 \end{aligned} \quad (56)$$

(56) says that the steady-state PITN is determined once the age 1 population growth rate, σ , the return rate of immigrants and the fertility rates of natives and immigrants are set. Conversely, we can choose σ so that the resulting PITN is consistent with the data¹⁶.

The capital stock at period t is the sum of the balance of individual savings and government savings. Let a_{t-1} be the balance of the government asset (or debt if it is negative) per capita at the end of period $t - 1$. Then, the total capital stock at period t is

$$K_t = \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1,i}^{i,j} s_{t-1}^{i,j} + a_{t-1} \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} \quad (57)$$

The (efficient unit) wage rate at time t , w_t and the pre-tax interest rate at time t , r_t , are determined as

$$w_t = (1 - \theta) K_t^\theta E_t^{1-\theta} L_t^{-\theta} \quad \text{and} \quad r_t = \theta K_t^{\theta-1} E_t^{1-\theta} L_t^{1-\theta} \quad (58)$$

Now, consider the initial balanced growth path where the capital to labor ratio (in efficient units) stays constant. Let $w^*(1 + \mu)^t$ and r^* be the wage rate and interest rate at period t on the initial balanced growth path. Let $s^{*i,j}(1 + \mu)^t$ and $b^{*i,j}(1 + \mu)^t$ be the savings and the social security benefit of type j of age i at period t on the initial balanced growth path. Let $a^*(1 + \mu)^t$ and $g^{*i,j}(1 + \mu)^t$ be the government asset(debt) per capita and the publicly provided private goods for type j of age i at period t on the initial balanced growth path. It is possible that immigrants use more public services, for example, children's education. Thus, I put superscript j . Let w^* and r^* be the efficiency unit wage rate and the interest rate on the initial balanced growth path determined from (57) and (58). Then the government budget constraint

¹⁶The RHS is an increasing function of σ at the parameter values that are consistent with the US. data.

on the initial balanced growth path is

$$\begin{aligned}
(1 + \mu)^{t-1} \left\{ \tau_{wt} w^* (1 + \mu) L_t + \tau_{rt} r^* \sum_{i=2}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s^{*i-1,j} + (1 + r^*) \sum_{i=2}^{80} \sum_{j=n,m} (1 - p_i) N_{t-1}^{i-1,j} s_{t-1}^{*i-1,j} \right. \\
- a^* (1 + \mu) \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} + (1 + r_t) a^* \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} - \sum_{i=1}^{80} \sum_{j=n,m} N_{t,i}^{i,j} g^{*i,j} (1 + \mu) \\
\left. - \sum_{i=46}^{80} \sum_{j=n,m} b^{*i,j} (1 + \mu) \times N_t^{i,j} - \kappa \sum_{i=46}^{80} \sum_{x=11}^{45} b^{*i,rm,x} (1 + \mu) \times N_t^{i,rm,x} \right\} = 0 \quad (59)
\end{aligned}$$

where τ_w^* and τ_r^* are the wage tax rate and capital income tax rate at the initial balanced growth path. $b^{*i,rm,x}$ and $N_t^{i,rm,x}$ are, respectively, the social security benefit and the number of immigrants of age i who returned to the home country at age x , but who are still eligible to claim social security benefits. κ is the share of those returned immigrants who actually claim social security benefits. The US social security does not require the residence as long as the individual is eligible for benefits. I assume that an immigrant is eligible to receive social security benefits as long as he or she pays social security contributions for at least 10 years. The first term of (59) is the revenue from wage tax and the second term is the revenue from capital income tax. The third term is the revenue from inheritance tax. In my simulation, I assume that the government imposes a 100 percent inheritance tax. The fourth term is the revenue from issuing government bonds and the fifth term is the expenditure on the principal and interest of the bonds. The sixth and seventh terms are the expenditure on the publicly provided private goods and the expenditure on social security benefits of natives and immigrants who reside in the host country. The last term is the social security benefits of immigrants who returned to the home country and who are eligible to receive social security benefit.

When the government increases the PITN, the wage rate decreases and the interest rate increases. To Pareto-improve welfare, I assume that the wage tax rate, interest tax rate and social security benefit are adjusted as in the theoretical analysis. More specifically, I assume that the government keeps the after-tax wage rate and interest rate at the same level as on the initial balance growth path. This implies that the

wage tax rate τ_{wt} and interest tax rate τ_{kt} are set as follows:

$$w_t(1 - \tau_{wt}) = w^*(1 + \mu)^t \times (1 - \tau_w^*) \text{ and } r_t(1 - \tau_{rt}) = r^*(1 - \tau_r^*) \quad (60)$$

The social security benefit is adjusted as follows:

$$b_{t-1+i}^{i,j} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45} (1 + \mu)^{45-i} w_{t-1+i} (1 - \tau_{wt}) l_{t-1+i}^{i,j} H^{i,j}}{(1 - \tau_w^*) \times 45 \times 12} \quad (61)$$

When the taxes and social security benefit formula are adjusted according to (60) and (61), the individual budget constraints is the same as on the initial balanced growth path¹⁷. Thus, consumption, labor supply and savings do not change. Then, there would be a surplus for the government budget even if the government were to spend the same amount of publicly provided private goods per person as at the initial balanced growth path, as analyzed in the previous section. The government can use this surplus to increase savings or to increase the level of publicly provided private goods. Let V be the distributional parameter that indicates what percentage of the budget surplus is put into savings. The government can keep increasing the government savings balance until the economy reaches the golden rule level or a modified golden rule level. As long as the MPK is greater or equal to the (modified) golden rule level, the balance of the government savings at period t is determined from the following equation:

$$(1 + \mu)^t a_t \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} = V \times SP_t$$

where SP_t is the government budget surplus at period t . If the MPK is equal to the

¹⁷Note that when $w_t(1 - \tau_{wt}) = w_t^*(1 - \tau_w^*)$, $AIME^j(t) = \frac{\sum_{i=1}^{45} (1 + \mu)^{45-i} w^*(1 + \mu)^{t-1+i} l_{t-1+i}^{i,j} H^{i,j}}{45 \times 12}$

(modified) golden rule level, V becomes 0. SP_t is defined as follows:

$$\begin{aligned}
SP_t = \tau_{wt} w_t L_t + (1 + \mu)^{t-1} \left\{ \tau_{rt} r_t \sum_{i=1}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s^{*i-1,j} + (1 + r_t) \sum_{i=1}^{80} \sum_{j=n,m} (1 - p_i) N_{t-1}^{i-1,j} s^{*i-1,j} \right. \\
(1 + r_t) \times a_{t-1} \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} - \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} \times g^{*,i,j} (1 + \mu) \\
\left. - \sum_{i=46}^{80} \sum_{j=n,m} b^{*,i,j} (1 + \mu) \times N_t^{i,j} - \kappa \sum_{i=46}^{80} \sum_{x=11}^{45} b^{*,i,rm,x} (1 + \mu) \times N_t^{i,rm,x} \right\} \quad (62)
\end{aligned}$$

The rest of the surplus that is available for increasing publicly provided private goods is $(1 - V) \times SP_t$. This is distributed equally to the entire population (including immigrants) at period t .¹⁸ Then, after increasing the PITN, the amount of publicly provided private goods, g_t^i , becomes

$$g_t^i = g^{*,i,j} (1 + \mu)^t + \frac{(1 - V) \times SP_t}{\sum_{i=1}^{80} (N_t^{i,n} + N_t^{i,m})} \quad (63)$$

For the (modified) golden rule, given the intergenerational discount rate for the modified golden rule, we say that the economy reaches the (modified) golden rule level if the marginal product of capital becomes equal to the sum of the growth rate of the efficiency unit of labor, the depreciation rate, and the inter generational discount rate for the modified golden rule. In the case of the golden rule, the intergenerational discount rate for the modified golden rule is set to zero.

4.2 Policy Experiments and Parameters Values for Simulation

Policy Experiments

For the percentage of immigrants to natives (PITN), I calculate the data from the US census at 2000. US Census 2000 shows that the PITN above the age of 20 is 15.5 percent. Thus, I assume that the PITN at the initial balanced growth path is

¹⁸The government could distribute this surplus more to the natives than to the immigrants. Thus, this assumption underestimates the welfare gain to the natives.

15.5 percent. For the target PITN, I look at past US data. Historically, the PITN was 5 percent in 1970 and increased to 18.3 percent in 2010.¹⁹ The PITN increased more than 10 percentage point within 40 years from 1970 to 2010, so I assume that a 10 percentage point increase in the PITN over 80 years is tolerable. Therefore, I set the target PITN at 25.5 percent. In the slow benchmark case, the PITN starts to increase from 15.5 percent and reaches the target PITN at the 80th year. After the 80th year, it remains on the same level. To see how accelerating the speed at which the PITN is increased affects my results, I also consider intermediate and fast cases, in which the PITN hits the target PITN at the 62nd or 42nd year.²⁰ I assume that all immigrants arrive at age 1. The graph for the PITN over time is shown in Figure 1.²¹

Fertility, Mortality and Return Rate of Immigrants and Other Parameter Values

Parameter values regarding fertility, mortality, return rate of immigrant, government expenditure, government debt, taxes, preferences and production function are quite standard. To save the space, I discuss those parameter values in Appendix A2.

¹⁹Data in 1970 is from the US Census. Data in 2010 is from the CPS. After 2000, data on the foreign-born are available only through the American Community Survey and the CPS, not the Census. For calculating the PITN, one concern is the comparability of the CPS and the Census data. In 2000 when both the census and the CPS are available, the PITN in the Census is 12.42% and 11.54% in the CPS. Thus, the difference between the Census and CPS would seem to be very minor. This view is shared by the Census bureau. Schmidley and Robinson (2003) conclude that the difference of the estimates based on the Census data and CPS data are trivial.

²⁰In the fast and intermediate cases, to allow smooth transition of the PITN, the PITN keeps increasing even after hitting the target PITN. See Figure 1.

²¹More specifically, each of the three cases is calculated as follows. Let $f_{15.5}$ be the steady-state age1 immigrant-native ratio when the PITN is 15.5 percent. Define $f_{25.5}$ in a similar fashion. Then, the age 1 immigrant-native ratio at period t , f_t , in the slow case is $f_t = f_{25.5}$ for all t . For the intermediate case, $f_t = f_{25.5} + 0.1 \times f_{25.5} \times (\frac{t}{15})$ for $1 \leq t \leq 15$, $f_t = 1.1 \times f_{25.5} - 0.1 \times f_{25.5} \times (t - 15)$ for $16 \leq t \leq 30$ and $f_t = f_{25.5}$ for $31 \leq t$. For the fast case, $f_t = f_{25.5} + 0.4 \times f_{25.5} \times (\frac{t}{15})$ for $1 \leq t \leq 15$, $f_t = 1.4 \times f_{25.5} - 0.4 \times f_{25.5} \times (t - 15)$ for $16 \leq t \leq 30$ and $f_t = f_{25.5}$ for $31 \leq t$.

Note that the steady-state age 1 immigrant-native ratio is defined in the first term of the RHS in equation (40). Thus, once the PITN is determined, the steady state age 1 immigrant-native ratio is calculated from equation (40).

4.3 Results

Figures A3, A4 and A5 in the appendices show the balance of assets, consumption and leisure for the life cycle of an individual at the initial balanced growth path in the benchmark analysis. At the age of 46, the consumption of leisure becomes 1 due to mandatory retirement. At the initial balanced growth path, the capital to output ratio is 2.98, which is higher than the value used in Storesletten (2.4) and Nishiyama and Smetter (2.7), but lower than the value used in the standard business cycle research, which is 3.2 (Cooley and Prescott(1995)). To check how my results are affected by the capital to output ratio, I change the time preference rate and examine how the results change for different values of the time preference rate in the robustness checks.

Table 1 shows the parameter values and the welfare effect of increasing the PITN for different values of V in equation (62), which is the share of the government surplus that is put into savings. As mentioned above, the initial PITN is set at 15.5 percent and the target INR at 25.5 percent. With respect to rate at which immigrants are accepted, I consider three cases in which it takes 80, 62 and 42 years for the PITN to reach the target PITN. Column (3) of Table 1 shows the share of the government surplus that is put into savings. Columns (4)-(9) are values that are calculated within the simulation. Column (4) shows how many years it takes for the economy to reach the (modified) golden rule level. When it does not reach the (modified) golden rule within 300 years, this is indicated by * or **. * indicates that the capital-to-labor ratio (efficient unit) at the 300th year is higher than at the initial balanced growth path, that it is increasing at the 300th year, but that it does not reach the (modified) golden rule level at the 300th year. ** indicates that the capital-to-labor ratio at the 300th year is lower than at the initial balanced growth path. For example, when $V = 100\%$, it takes 112 years to reach the golden rule level in the slow acceptance case(row (16)). Column (5) shows how much the capital stock per efficient unit of labor increases at the golden rule compared with the level at the initial balanced growth path. When $V = 100\%$, it shows that the capital stock per efficient unit of labor increases by 102 percent. Column (6) shows how much the publicly provided private goods will increase compared with the level at the initial balanced growth path. At $V = 100\%$,

column (6) shows that the publicly provided private goods will increase by about 36 percent. To calculate columns (5) and (6), I evaluate at the year at which the economy reached the golden rule level if it reaches that level within 300 years, and at the 300th year otherwise. Column (7) shows the percentage by which the utility, measured by the expenditure function, of the cohort that is born when the economy reaches the golden rule level increases compared with the utility of the same cohort if it were on the initial balanced growth path. When the economy does not reach the golden rule level, I calculate the utility of the cohort that is born at the 300th year. In the expenditure function, the price vector on the initial balanced growth path is used for evaluating the utility. Note that all welfare gain is distributed through an increase in publicly provided private goods. Column (8) shows the share of the present discounted value (PDV) of the increased publicly provided private goods in the initial GDP. For example, in row (16), the PDV of the increased publicly provided private goods is 12 percent of the initial GDP. For discounting the increased publicly provided private goods in future periods, I apply a 5 percent discount rate instead of the equilibrium interest rate, so that I can eliminate the effect of the discount rate when comparing different cases. Column (9) measures the welfare gain of natives and their descendants, not that of immigrants or their descendants. It measures how much the utility of natives and their descendants, not including immigrants and their descendants, is Pareto-improved by accepting more immigrants compared with the utility level at the initial balanced growth path. It evaluates the utility by using the expenditure function. I apply the price vector at the initial balanced growth path for the expenditure function and the equivalent variation to measure the difference of the levels of utility in two cases. To discount the welfare gain of future cohorts, I again use a 5 percent discount rate. Note that in all cases in Table 1, I assume that the government does not discriminate between immigrants and natives with respect to publicly provided private goods. Both natives and immigrant consume publicly provided private goods equally. Column (9) in row (16) shows that the PDV of the Pareto-improving welfare gain of natives and their descendants comprises 11 percent of the initial GDP.

In rows (2)-(3), (5)-(6), and (8)-(9)...., I shorten the years needed to reach the target PITN and increase the speed with which the PITN increases. When the number of years needed to reach the target PITN is reduced to 42, instead of 80, and $V=100$, the PDV of the welfare gain of increasing the PITN comprises 13 percent of the initial GDP (row (18)).

Table 1 shows that the number of years taken to reach the golden rule level decreases as V increases, because the government saves more for future cohorts. On the other hand, the PDV of the increased utility, measured as the share of the initial GDP, increases as V decreases as long as the V is greater than or equal to 50%. The PDV of the increased utility, measured as the share of the initial GDP, is highest when $V=50\%$. In this case, the quantified Pareto improvement ranges from 21% to 26% of the initial GDP.

In Table 1, I set the intergenerational discount rate for the modified golden rule at 0% and set the target level of the capital stock at the golden rule level. However, the targeting capital stock at the golden rule level does not necessarily maximize the PDV of the welfare gain. In Table A2, I examine the effect on welfare of increasing the PITN for different target levels of capital stock by changing the value of the intergenerational discount rate for the modified golden rule when $V=100\%$. Table A2 shows that the welfare gain is maximized when the intergenerational discount rate is set at 3 percent.

In Table 2, which I consider as the representative case of my simulation, I recalculate the all rows of Table 1 by setting the intergenerational discount rate for the modified golden rule at 3 percent. In Table 2, the Pareto-improving welfare gain of increasing the PITN is more than 20 percent of the initial GDP and the capital stock per efficient unit of labor increases by 18 percent as long as V is greater than or equal to 50 percent.

Figure 2 and Figure A6 show the marginal product of capital and the capital to output ratio over time for different values of V when the target capital stock is the modified golden rule level (3% intergenerational discount rate). The marginal product of capital increases initially, due to the acceptance of more immigrants. However, as

the government savings balance increases, the capital stock per efficient unit of labor starts to increase and the marginal product of capital continues to decrease until the economy reaches the golden rule level. The capital to output ratio also displays a similar pattern.

Figure 3 compares the utility level of all cohorts on the initial balanced growth path with the utility level of all cohorts for different values of the share for the government savings (V) when the target capital stock is set at the golden rule level. For example, when $V=100\%$, all the surplus is put into savings until the economy reaches the golden rule level and the surplus is distributed to individuals only after the economy reaches the golden rule level. This implies that the utility of the 65th cohort, which dies at the 65th year, starts to experience higher utility than at the initial balanced growth path.

In all cases considered in Tables 1,2,3 and 4, the results of simulations show that all cohorts are Pareto-improved; this confirms my theoretical results.²²

Figure 4 shows the importance of government savings balance at the new equilibrium path. It calculates the ratio between the interest income from the government savings balance and the social security benefit payment in each period. When the economy reaches the golden rule level, the interest income from the government savings balance comprises 70 percent of the social security benefit payments. Thus, at the new equilibrium path, the interest income from the government savings balance is contributing substantial amount.

²²The results of this simulation differ from those of some of the previous studies. For example, Kotlikoff et.al reports that the welfare gain of doubling immigrants is very small. Lee and Miller argues that the fiscal impact of accepting additional 100,000 immigrants is very small. Several factors in those studies generate different results. Kotlikoff et.al analyze immigration policy in an open economy setting, whereas I use a closed economy setting. In an open economy setting, the effect of additional government savings is offset by the mobility of capital. In my simulation, the government can use the budget surplus, which is obtained by accepting more immigrants, for savings and the amount of savings affects the overall welfare. Lee and Miller consider a much smaller increase in the number of immigrants than I do. On the other hand, my simulation results are consistent with Auerbach Oreopoulos (2000). They find that if the initial fiscal imbalance is not adjusted, then the welfare loss of halving the number of new immigrants is substantial. This is consistent with my theoretical and simulation results.

4.4 Robustness Checks

Table 3 and 4 present results of robustness checks of Table 2. Rows (1)-(6) in Table 3 check whether the results in Table 2 are sensitive to the initial government debt (asset) level. As I argued above, different authors assume different levels for the government debt (asset) at the initial balanced growth path. In rows (1)-(3) in Table 5, I set the initial government debt to 10 % of the private capital instead of 0 %. In rows (4)-(6), I assume that the initial government debt level is -10% of the private capital.

Rows (7)-(12) in Table 3 check whether the results in Table 2 are sensitive to the replacement rate. The theoretical analysis implies that higher intergenerational redistribution will result in the acceptance of more immigrants yielding a higher welfare gain. Thus, it is predicted that as the replacement rate decreases, so does the welfare gain of accepting more immigrants. Rows (7)-(12) confirms this prediction. Decreasing the replacement rate 10 percentage points decreases the welfare gain by 7 percentage point in terms of the percentage of the initial GDP.

Rows (13)-(21) in Table 3 conduct sensitivity checks on the immigrants' level of earnings. Following Storesletten (1995), I set the immigrant wage rate at 84.3 percent of that of natives. In my calculation using the CPS 2000 June supplement, I found that immigrants' earning are 91 percent of natives' earnings. Rows (13)-(15) assume that the wage rate of immigrants is 89.3 percent of that of natives, rather than 84.3 percent. Rows (16)-(18) assume that the wage rate of immigrants is 79.3 percent of that of natives. Rows (19)-(21) assume that the wage rate of the immigrants is 75 percent of that of natives. The results in rows (19)-(21) show that if immigrants earn 25 percent lower than natives, the welfare gain of accepting more immigrant is 17 percent of the initial GDP, instead of 23 percent of the initial GDP.

Rows (22)-(27) in Table 3 conduct sensitivity checks on the consumption by immigrants of publicly provided private goods. In Table 1 and 2, I assumes that immigrants and native consume publicly provided private goods equally as the previous empirical studies indicate. In rows (22)-(24), I assume that young immigrants consume 20 percent more publicly provided private goods than young natives. The PDV of the welfare gain comprises 17 percent of the initial GDP instead of 23 percent. Rows

(25)-(27) assume that immigrants of all ages consume 20 percent more publicly provided private goods than natives. In this case, the PDV of the welfare gain comprises 15 percent of the initial GDP.

In Table 4, I conduct robustness checks by changing the values of parameters of the utility function and the return rate of immigrants. Rows (1)-(6) of Table 4 examine the sensitivity of the results regarding the values of the parameters for constant relative risk aversion (CRRA). In Table 1-3, I assume that the CRRA is equal to 3. Auerbach and Kotlikoff, and Storesletten assume that the CRRA=4, while Nishiyama and Smetter assume that the CRRA is 2. Rows (1)-(3) assume that the CRRA is equal to 4 and rows (5)-(6) assume that it is 2. The results presented on rows (1)-(6) show that the results presented in Table 2 is not sensitive to the value of CRRA. Rows (7)-(12) examine the sensitivity of the results to the time preference rate. Although it is quite common to assume that the time preferences rate is greater than 1, readers might think that the results presented in Table 1 and 2 can be sensitive with respect to the assumption that the time preference rate is greater than 1. Theoretically, lowering the time preference rate will result in lowering savings and will decrease the capital to labor ratio at the initial balanced growth path. Since increasing the number of immigrants will increase the capital stock and the production function exhibits diminishing marginal product of capital, lowering the initial capital stock as a result of assuming a lower time preference rate will increase the welfare gain of increasing the PITN. Rows (7)-(12) confirm this theoretical prediction, but they show that the magnitude of those changes is very small. For example, changing the time preference rate from 1.011 to 0.99 increase the PDV of the welfare gain by only 1 percentage point.

Rows (13)-(15) in Table 4 conduct robustness checks by assuming that the return rate of immigrants is equal to 0. Rows (13)-(15) show that the results presented in Table 2 do not change so much and that they are robust regarding the return rate of immigrants.

4.5 Summary of the Simulation Results

The simulation results presented in Tables 1-4 can be summarized as follows. In the case of slow acceptance of more immigrants, in which the PITN increases from 15.5 percent and reaches 25.5 percent at the 80th year, the PDV of the increased utility of the natives and their descendants, measured by the expenditure function, is 23 percent of the initial GDP when the target capital stock is the modified golden rule level (3 % intergenerational discount rate). The economy reaches the modified golden rule level at 65th year and the capital stock per efficient unit of labor increases by 18 percent. In the fast acceptance case, in which the PITN reaches 25.5 percent at the 42th year, the PDV of the welfare gain comprises 28 percent of the initial GDP and the economy reaches the modified golden rule level at 59th year.

For robustness checks, I conduct sensitivity checks for changing the values of the following parameters: initial government debt (asset) level, the replacement rate, level of earnings of immigrants, immigrants' consumption of publicly provided private goods, the CRRA, the time preference rate and probability of the return to home country by immigrants. The sensitivity checks show that the results are quite robust.

5 Conclusion

I have examined, both theoretically and quantitatively, the effect on welfare of increasing the PITN in the presence of a PYGO social security system. The results for both the theoretical quantitative analysis show that the welfare gain of accepting more immigrants is robust and non-trivial. The PDV of the welfare gain of increasing the PITN from 15.5 to 25.5 percent amounts to about 20 percent of the initial GDP. In the shortest case, the economy reaches the golden rule level at the 112th year in a Pareto-improving way. The analysis suggests that accepting more immigrants can be an important tool for policy makers when addressing the economic problems caused by a PYGO social security system.

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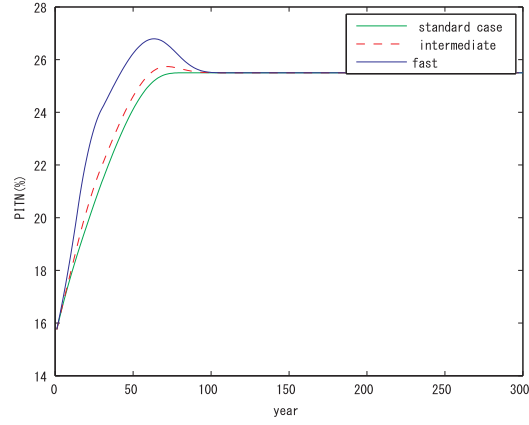


Figure 1: PITN over time. The initial PITN is set at 15.5% and the target PITN is set at 25.5%.

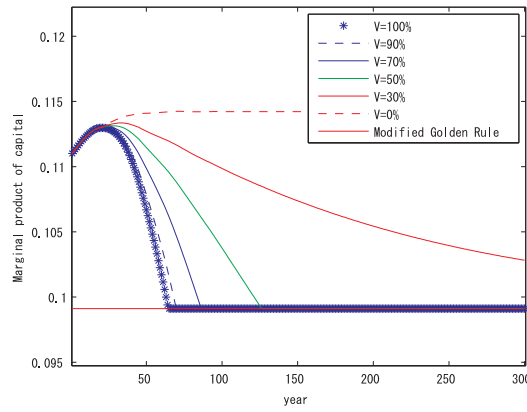


Figure 2: The marginal product of capital over time for different values of the share of the surplus used for government savings (V). The target capital stock is the modified golden rule level with 3 % intergenerational discount rate.

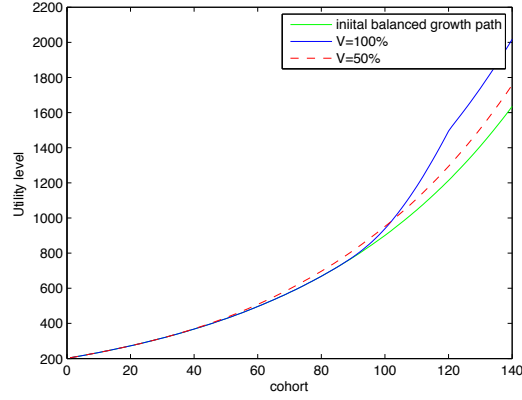


Figure 3: Utility level of different cohorts for different values of the share of the surplus used for government savings (V). The target capital stock level is the modified golden rule level with 3% intergenerational discount rate for the modified golden rule.

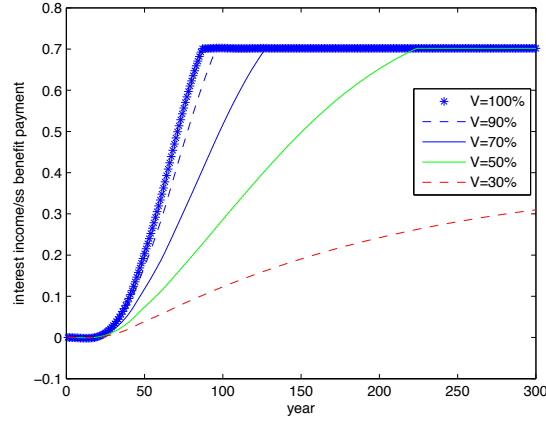


Figure 4: The ratio between the interest income from the government savings balance and the social security benefit payment in each period. It is assumed that the PITN reaches the target PITN at 80th year. The inter-generational discount rate for the modified golden rule is set at 3%.

Table 1
The effect of increasing the PITN
(The target capital stock is the golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	share of the surplus for the gov. savings (V)	year reaching the golden rule	% increase of capital stock per efficient unit labor at the golden rule	%change of publicly provided private goods per capita at the golden rule	% change of welfare of cohort born at the golden rule	share of the sum of the PDV of increased publicly provided private goods in the initial GDP	share of the sum of PDV of welfare gain of all natives and their descendants in the initial GDP
share of the surplus for the gov. savings =0%								
1	80	0%	300**	-4.37%	5.00%	0.53%	15.57%	17.72%
2	62	0%	300**	-4.37%	5.00%	0.53%	16.74%	18.94%
3	42	0%	300**	-4.37%	5.00%	0.53%	20.20%	22.47%
share of the surplus for the gov. savings =30%								
4	80	30%	300*	11.89%	14.69%	1.41%	17.86%	19.59%
5	62	30%	300*	12.00%	14.76%	1.42%	19.20%	20.94%
6	42	30%	300*	73.43%	30.82%	2.88%	25.32%	26.09%
share of the surplus for the gov. savings =50%								
7	80	50%	300*	71.21%	30.27%	2.79%	19.66%	20.79%
8	62	50%	300*	71.82%	30.42%	2.81%	21.10%	22.18%
9	42	50%	300*	73.43%	30.82%	2.88%	25.32%	26.09%
share of the surplus for the gov. savings =70%								
10	80	70%	184	102.43%	35.86%	3.77%	18.48%	18.92%
11	62	70%	182	102.43%	35.86%	3.77%	19.75%	20.09%
12	42	70%	119	50.99%	30.14%	3.17%	25.87%	25.56%
share of the surplus for the gov. savings =90%								
13	80	90%	127	102.43%	35.86%	3.77%	14.52%	13.75%
14	62	90%	125	102.43%	35.86%	3.77%	15.47%	14.56%
15	42	90%	120	102.43%	35.86%	3.77%	18.21%	16.80%
share of the surplus for the gov. savings =100%								
16	80	100%	112	102.43%	35.86%	3.77%	12.23%	10.69%
17	62	100%	110	102.43%	35.86%	3.77%	13.04%	11.33%
18	42	100%	106	102.43%	35.86%	3.77%	15.38%	13.14%

Notes

1. In all rows, the initial PITN is 15.5% and target PITN is 25.5%. The replacement rate is 60 %, CRRA=3 and the time preference rate is 1.011. The equilibrium capital to output ratio on the initial balanced growth path is 2.98.
2. In all rows, wage rate of immigrants is 84.3 % of that of natives.
3. * indicates that the capital stock per efficient unit labor does not reach the golden rule level within 300 years. Its value at the 300th year is higher than at the initial balanced growth path and keeps increasing at the 300th year. The percent change of the capital stock per efficient unit of labor is evaluated at the 300th year.
4. ** indicates that capital stock per efficient unit labor does not reach the golden rule level within 300 years and the capital stock per efficient unit labor at the 300th year is lower than at the initial balanced growth path. The percent change of the capital stock per efficient unit of labor is evaluated at 300th year.

Table 2
The effect of increasing the PITN for different values of V
(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	share of the surplus for the gov. savings (V)	year reaching the golden rule	% increase of the capital stock per efficient unit labor at the golden rule	% change of publicly provided private goods per capita at the golden rule	% change of welfare of cohort born at the golden rule	share of the PV of increased publicly provided private goods in the initial GDP	share of the PDV of welfare gain of all natives and their descendants in the initial GDP
share of the surplus for the gov. savings =0%								
1	80	0%	300**	-4.37%	5.00%	0.53%	15.57%	17.72%
2	62	0%	300**	-4.37%	5.00%	0.53%	16.74%	18.94%
3	42	0%	300**	-4.37%	5.00%	0.53%	20.20%	22.47%
share of the surplus for the gov. savings =30%								
4	80	30%	300*	11.89%	14.69%	1.41%	17.86%	19.59%
5	62	30%	300*	12.00%	14.76%	1.42%	19.20%	20.94%
6	42	30%	300*	12.32%	14.96%	1.45%	23.18%	24.80%
share of the surplus for the gov. savings =50%								
7	80	50%	127	18.19%	18.51%	1.95%	19.86%	21.06%
8	62	50%	124	18.19%	18.51%	1.95%	21.31%	22.47%
9	42	50%	116	18.19%	18.51%	1.95%	25.58%	26.45%
share of the surplus for the gov. savings =70%								
10	80	70%	87	18.19%	18.51%	1.95%	21.31%	22.11%
11	62	70%	78	18.19%	18.51%	1.95%	27.15%	27.49%
12	42	70%	78	18.19%	18.51%	1.95%	27.15%	27.49%
share of the surplus for the gov. savings =90%								
13	80	90%	70	18.19%	18.51%	1.95%	22.32%	22.85%
14	62	90%	68	18.19%	18.51%	1.95%	23.81%	24.24%
15	42	90%	63	18.19%	18.51%	1.95%	28.14%	28.13%
share of the surplus for the gov. savings =100%								
16	80	100%	65	18.19%	18.51%	1.95%	22.71%	23.14%
17	62	100%	63	18.19%	18.51%	1.95%	24.19%	24.51%
18	42	100%	59	18.19%	18.51%	1.95%	28.51%	28.37%

Notes

1. In all rows, the initial PITN is 15.5% and target PITN is 25.5%. The replacement rate is 60 %, CRRA=3 and the time preference rate is 1.011. The equilibrium capital to output ratio on the initial balanced growth path is 2.98.
2. In all rows, wage rate of immigrants is 84.3 % of that of natives.
3. The intergenerational discount rate for the modified golden rule is 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the intergenerational discount rate for the modified golden rule.
3. * indicates that the capital stock per efficient unit labor does not reach the modified golden rule level within 300 years. Its value at the 300th year is higher than at the initial balanced growth path and keeps increasing at the 300th year. The percent change of capital stock per efficient unit of labor is evaluated at the 300th year.
4. ** indicates that the capital stock per efficient unit labor does not reach the modified golden rule level within 300 years and the capital stock per efficient unit labor at the 300th year is lower than at the initial balanced growth path. The percent change of the capital stock per efficient unit of labor is evaluated at the 300th year.

Table 3
Robustness checks
The role of the initial government debt level, the replacement rate, immigrants earnings and the use of public services by immigrants
(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	capital to output ratio at the initial balanced growth path	years taken to reach the modified golden rule level	% increase of capital stock per efficient unit labor at the new balanced growth path	%change of publicly provided private goods per capita at the new balanced growth path	% change of welfare of the cohort born at the new balanced growth path	share of the sum of PDV of increased publicly provided private goods in the initial GDP	share of the sum of the PDV of welfare gain of all natives and their descendants in initial GDP
initial government debt ratio (% of private capital)= 10 %								
1	80	2.91	63	22.39%	21.87%	2.29%	28.48%	27.22%
2	62	2.91	61	22.39%	21.87%	2.29%	30.31%	28.82%
3	42	2.91	57	22.39%	21.87%	2.29%	35.66%	33.30%
initial government debt ratio(% of private capital)= -10 %								
4	80	3.04	67	14.71%	15.75%	1.67%	18.30%	19.68%
5	62	3.04	65	14.71%	15.75%	1.67%	19.51%	20.87%
6	42	3.04	61	14.71%	15.75%	1.67%	23.01%	24.16%
replacement rate =0.55								
7	80	3.03	68	14.87%	15.79%	1.67%	18.05%	19.32%
8	62	3.03	66	14.87%	15.79%	1.67%	19.24%	20.48%
9	42	3.03	61	14.87%	15.79%	1.67%	22.67%	23.70%
replacement rate =0.5								
10	80	3.1	71	11.51%	13.07%	1.39%	13.68%	15.38%
11	62	3.1	69	11.51%	13.07%	1.39%	14.572%	16.30%
12	42	3.1	65	11.51%	13.07%	1.39%	17.159%	18.84%
level of human capital of immigrants is 89.3 % of that of natives								
13	80	2.98	62	18.20%	18.96%	2.01%	25.22%	25.93%
14	62	2.98	60	18.20%	18.96%	2.01%	26.871%	27.48%
15	42	2.98	56	18.20%	18.96%	2.01%	31.702%	31.86%
level of human capital of immigrants is 79.3 % of that of natives								
16	80	2.98	69	18.18%	18.05%	1.89%	20.09%	20.24%
17	62	2.98	67	18.18%	18.05%	1.89%	21.392%	21.43%
18	42	2.98	62	18.18%	18.05%	1.89%	25.165%	24.74%
level of human capital of immigrants is 74.3 % of that of natives								
19	80	2.98	73	18.18%	17.58%	1.83%	17.32%	17.20%
20	62	2.98	71	18.18%	17.58%	1.83%	18.430%	18.20%
21	42	2.98	67	18.18%	17.58%	1.83%	21.629%	20.96%
young adult immigrants consume publicly provided private goods 20% higher than young adult natives								
22	80	2.97	74	18.46%	17.70%	1.90%	17.27%	17.01%
23	62	2.97	72	18.46%	17.70%	1.90%	18.343%	17.96%
24	42	2.97	68	18.46%	17.70%	1.90%	21.409%	20.56%
immigrants of all ages consume publicly provided private goods 20% higher than natives								
25	80	2.96	77	18.84%	16.89%	1.86%	15.71%	15.25%
26	62	2.96	75	18.84%	16.89%	1.86%	16.674%	16.09%
27	42	2.96	71	18.84%	16.89%	1.86%	19.410%	18.36%

Notes

1. In all rows, V is 100 percent and the inter-generational social discount factor for the modified golden rule is set at 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.
2. In all rows, the initial PITN is 15.5% and the target PITN is 25.5%. CRRA=3 and the time preference
4. The wage rate of immigrants is 84.3 % of that of natives in all rows except in rows (13) to (18).
5. Immigrants consume the same amount of publicly provided private goods as natives in rows (22) to (27).
6. The replacement is 0.6 in all rows except rows (7) to (12).

Table 4
Robustness checks (2)

The role of the CRRA, the time preference rate and the return rate of immigrant

(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target INR	capital to output ratio at the initial balanced growth path	years taken to reach the modified golden rule level	% increase of the capital stock per efficient unit labor at the new balanced growth path	%change of publicly provided private goods per capita at the new balanced growth path	% change of welfare of cohort born at the new balanced growth path	share of the sum of the PDV of increased publicly provided private goods in the initial GDP	share of sum of the PDV of welfare gain of all natives and their descendants in initial GDP
CRRA=4								
1	80	2.634	74	41.87%	33.40%	3.28%	32.25%	22.18%
2	62	2.634	72	41.87%	33.40%	3.28%	34.04%	23.26%
3	42	2.634	69	41.87%	33.40%	3.28%	39.10%	26.15%
CRRA=2								
4	80	3.44	63	11.52%	12.70%	1.47%	16.26%	25.82%
5	62	3.44	61	11.52%	12.70%	1.47%	17.52%	27.70%
6	42	3.44	56	11.52%	12.70%	1.47%	21.32%	33.23%
the time preference rate=1								
7	80	2.67	72	39.41%	32.00%	3.17%	32.35%	23.15%
8	62	2.67	71	39.41%	32.00%	3.17%	34.184%	24.31%
9	42	2.67	67	39.41%	32.00%	3.17%	39.396%	27.43%
the time preference rate=0.99								
10	80	2.42	74	61.43%	46.18%	4.38%	43.96%	23.52%
11	62	2.42	73	61.43%	46.18%	4.38%	46.130%	24.51%
12	42	2.42	70	61.43%	46.18%	4.38%	52.171%	27.09%
the retrun rate of immigrant is 0%								
13	80	2.97	63	17.04%	17.86%	1.87%	24.00%	24.25%
14	62	2.97	61	17.04%	17.86%	1.87%	25.56%	25.68%
15	42	2.97	61	14.87%	15.79%	1.67%	22.67%	23.70%

Notes

1. In all rows, V is 100 percent. The parameter values of the wage rate of immigrants, the replacement rate, and the consumption of publicly provided private goods by immigrants are the same as in Table 2.
2. In all rows, the initial PITN is 15.5% and the target PITN is 25.5%.
3. The inter-generational discount rate for the modified golden rule is 3% except in rows (4) to (6). In rows (4) to (6), the economy's capital stock is already above the modified golden rule level with a 3% inter-generational discount rate. In rows (4) to (6), I set the inter-generational discount rate at 2% instead of 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.

Appendices A (The following appendices are put on the Journal's webpage as supplement material, but not included in the main paper.)

Appendix A1

Consider a simple case in which the depreciation rate is 100% ($\delta = 1$) and immigration policy α is implemented at the initial steady state. Assume that the population of the old is equal to one. At period 0, the following resource constraint must hold:

$$F(R(\alpha^*)l^*, s^* + a^*) - (c^{o*} + g^o + g^{ind}) - R(\alpha^*)(c^{y*} + s^* + a^* + g^y + g^{ind}) = 0. \quad (64)$$

Note that $F(R(\alpha^*)l^*, s^* + a^*)$ is the GDP per one old individual when one plus the population growth rate is equal to $R(\alpha^*)$. Now consider the graph of $(y, R(\alpha))$ where the vertical axis measures the GDP per one old individual and the horizontal axis measures one plus the population growth rate which is $R(\alpha)$. This implies that $y = F(R(\alpha)l^*, s^* + a^*)$. Next, draw another graph defined by $(y, R(\alpha))$ where $y = (c^{y*} + s^* + a^* + g^y + g^{ind})R(\alpha)$. This is the line that passes through the origin and whose slope is $c^{y*} + s^* + a^* + g^y + g^{ind}$. At $R(\alpha) = R(\alpha^*)$, the vertical distance of this line represents the total amount of resources used for the young divided by the number of old individuals when one plus the population growth rate is equal to $R(\alpha^*)$. Thus, the difference between $y = F(R(\alpha)l^*, s^* + a^*)$ and $y = (c^{y*} + s^* + a^* + g^y + g^{ind})R(\alpha)$ at $R(\alpha) = R(\alpha^*)$ represents the amount of resources used for one old individual at the initial steady state.

Now suppose that a social planner increases the population growth rate by accepting more immigrants permanently. This implies that $R(\alpha)$ increases by $R'(\alpha^*)$ from $R(\alpha^*)$. If the slope of $y = F(R(\alpha)l^*, s^* + a^*)$ at the $R(\alpha) = R(\alpha^*)$ is greater than $c^{y*} + s^* + a^* + g^y + g^{ind}$, the social planner can maintain the same allocation of resources per each young individual (private consumption, savings, government provided private goods and age-independent public goods) and increase the alloca-

tion of resources to each old individual. Clearly this constitutes Pareto improvement. Note that when the government accepts immigrants there is a surplus that is equal to $R'(\alpha^*)\{l^*F_L((R(\alpha)l^*, s^* + a^*) - (c^{y*} + s^* + a^* + g^y + g^{ind}))\}$ in every period(see Figure A1).

Appendix A2

Fertility, Mortality and Return Rate of Immigrants

For the age-nativity specific fertility, I use the CPS 2000 June supplement, which the census bureau also uses for calculating on the age-nativity specific fertility. Figure A2 in the appendices shows the average of the total number of births by each woman's age and it shows that immigrant women have a greater total number of births than native women at all ages. From this figure, I calculate $\eta^{i,n}$ and $\eta^{i,m}$, the age-specific birth rate for native and immigrant women. This is shown in Table A1. For the adult mortality profile, p_i , I take the values from Nishiyama and Smetter (2007). I set d , the sum of infant and child mortality, 1.7 percent, using the Vital Statistics of the US for 1993. For the return rate of immigrants to their home countries, I use the official census estimate that was conducted by Ahmed and Robinson (1994). They estimate that for the first 10 years, second 10 years and the third 10 years, the return rate of immigrants is 19 percent, 9 percent and 7 percent, respectively. For annual rate, those numbers correspond to 2.085 percent, 0.938 percent and 0.723 percent, respectively. Thus, I set $\hat{p}_i = 0.9715$ for $2 \leq i < 10$, $\hat{p}_i = 0.99062$ for $11 \leq i \leq 20$, $\hat{p}_i = 0.9927$ for $21 \leq i \leq 30$ and $\hat{p}_i = 1$ for $i \geq 31$.

Once the age-nativity specific fertility rate, infant-child mortality and the initial PITN are set, then the annual population growth rate is calculated automatically according to equation (56) with the assumption that the PITN is at the steady state. With the estimated age-nativity specific fertility, the infant-child mortality and an initial PITN of 15.5 percent, the annual growth rate of the population of age 1 becomes 0.39 percent.²³ This implies that at the initial steady state, the government accepts

²³The annual CPS data on immigrants and natives from 1995 to 2010 shows that the median annual growth rate of the total population(sum of natives and immigrants) aged from 20 to 40 is

immigrants such that the annual growth rate of immigrant of age 1 becomes 0.39 percent.

Age-nativity Specific Government Expenditure

I assume that the age-specific government expenditure, $g^{*i,j}$ is the same for natives and immigrants. Empirical studies show that there is no systematic difference in the use of public services by the two groups.²⁴ Thus, I assume that for $1 \leq i \leq 24$, $g^{*i,j} = g^y$, for $25 \leq i \leq 44$, $g^{*i,j} = g_m$ and for $45 \leq i \leq 80$, $g^{*i,j} = g_o$. I assume that g^y, g^m and g^o are 24.5%, 13.4% and 23.2 % of GDP per capita at the initial balanced growth path following Storesletten (1995) and Auerbach, Kotlikoff Hagememann and Nicoletti (1989). On the other hand, to check the robustness of my results, I also assume that immigrants consume 20 percent more publicly provided private goods than natives in the robustness checks.

Utility Function, Production Function and Human Capital Profile

A precise estimate for the coefficient for relative risk aversion, γ , has not yet been found in the literature. Auerbach and Kotlikoff (1986) and Storesletten (2000) assumed that γ is 4. Nishiyama and Smetter (2007) set γ equal to 2. I assume that $\gamma = 3$ and check the robustness with $\gamma = 2$ and $\gamma = 4$. For the time preference rate, β , following Hurd (1989) and Storesletten (2000), I assume that $\beta = 1.011$. Higher β implies higher savings and a higher capital to output ratio. To check the sensitivity of my results, I also calculate with $\beta = 1$ and $\beta = 0.99$. I assume that the leisure share in the utility function, ζ , is 0.33.

0.12% while the median growth rate of immigrants aged from 20 to 40 is 1.85%. In the theoretical model, the model assumes that the age 1 population growth rate of immigrants and natives are the same. Thus, the theoretically predicted growth rate of age 1 population, 0.39%, is between the growth rate of immigrants aged 20-40 and the growth rate of the total population aged 20-40.

²⁴Borjas and Hilton (1996) show that immigrants have a higher participation rate in welfare programs than natives. Fix, Passel, and Zimmermann (1996) show that these differences are explained by the higher participation rate in welfare programs among refugees and retired immigrants and that there is no differences among labor immigrants. Thus, for the theoretical part, I assume that g^i is independent of the place of birth. For the computational part, I relax this assumption for the robustness checks.

I assume that the depreciation rate of capital, δ , is equal to 0.047. For the capital share in the production function, θ , I set $\theta = 0.4$. For technological progress, I assume that income per capita growth rate, μ , is 0.015.

For the human capital profile of natives, H_i^s , I take the value from Auerbach and Kotlikoff.

$$H^{i,n} = \exp(4.47 + 0.033 \times i - 0.00067 \times i^2) \quad \text{for } 1 \leq i \leq 45 \quad (65)$$

$$H^{i,n} = 0 \quad \text{for } 46 \leq i \quad (66)$$

For the human capital profile of the immigrants, Storesletten (1995) showed that immigrants' earnings are, on average, 15.7 percent lower than those of natives.²⁵ Similarly, using the CPS June 2000 supplement, my calculations show that immigrants' earnings are 10 percent lower than those of natives. Using these estimates as a basis, I assume that the efficient unit of human capital of immigrants is 84.3 percent of that of natives and that $H^{i,m} = 0.843 \times H_i^n$ in the benchmark calculation. To examine the robustness of my results, I change the level of human capital from 84.3 percent to 89.3 percent, 79.3 percent or 74.3 percent and re-check the results.

Taxes and Government Debt

For capital income tax, I take the value from Nishiyama and Smetter (2007) and assume that $\tau_k = 0.28$. For the level of social security benefit, a higher replacement rate means that greater intergenerational redistribution of income, which leads to greater welfare gain as a result of increasing the immigrant population. Following Auerbach and Kotlikoff, in the benchmark case I set the replacement 0.6 and check the robustness of my results by varying it from 0.6 to 0.55 and 0.5.

As for the initial level of government debt or assets, different authors set different levels. Storesletten considered only government debt and assumed that the initial level is 50 percent of the initial GDP. With his estimate of the initial capital to output

²⁵Figure 2.2 of Storesletten (1995) shows that at age 20, 25, 30, 35, 40, 45, the wage rate of immigrants is lower than that of natives by 15%, 20%, 17.8%, 16.4%, 12% and 13% respectively. By averaging those rates, I obtained a working value of 15.7%.

ratio being 2.4, his assumption implies that government debt is about 20 percent of private capital. Nishiyama and Smetter considered not only government debt, but also government assets by using the BEA information on the government's fixed capital. They assumed that at the initial steady state, the government has positive net assets of 10 percent of the total private capital. This naturally leads to a higher capital to output ratio at the initial balanced growth path. Herein, as a benchmark case, I assume that initial level of government debt or assets is 0 percent of the private capital and I experiment with the assumption that the government debt is 10 percent or minus 10 percent of the private capital.

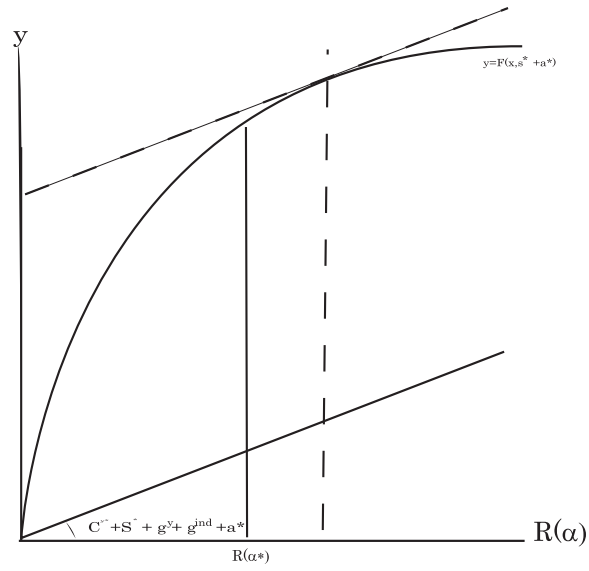


Figure A1– The one plus population growth rate and the resources used for one old person. The curve through the origin is $y = F(R(\alpha), s^* + a^*)$ and the straight line is $y = (c^{y*} + s^* + g^y + g^{ind} + a^*)R(\alpha)$. The vertical distance of the straight line through the origin measures the total resources used for young divided the number of old people. The difference between the curved line and the straight line measures the resource used per one old person at $R(\alpha)$. On the above graph, increasing $R(\alpha)$ will increase the resources available per one old person without decreasing the resources used for the young.

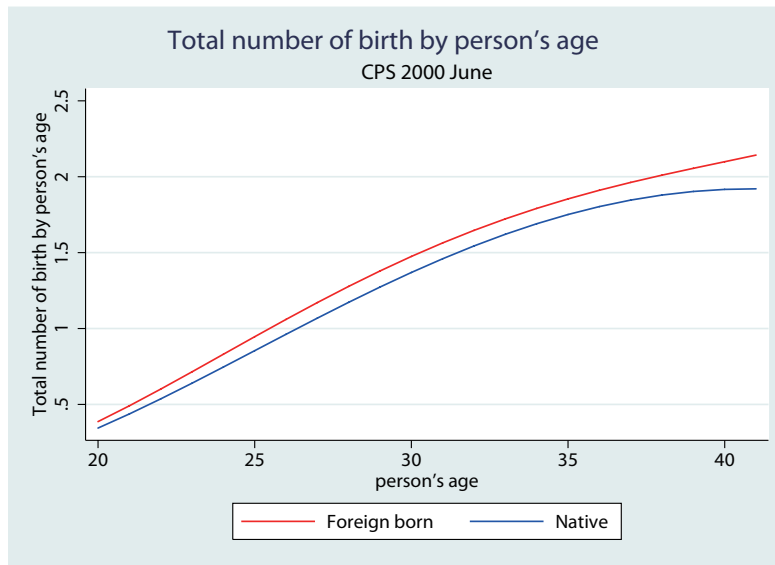


Figure A2—Total number of births by each age. The data source is the CPS 2000 supplement. The total number of births by each age is regressed on a sixth-order polynomial function of age separately for natives and immigrants. The predicted values are plotted.

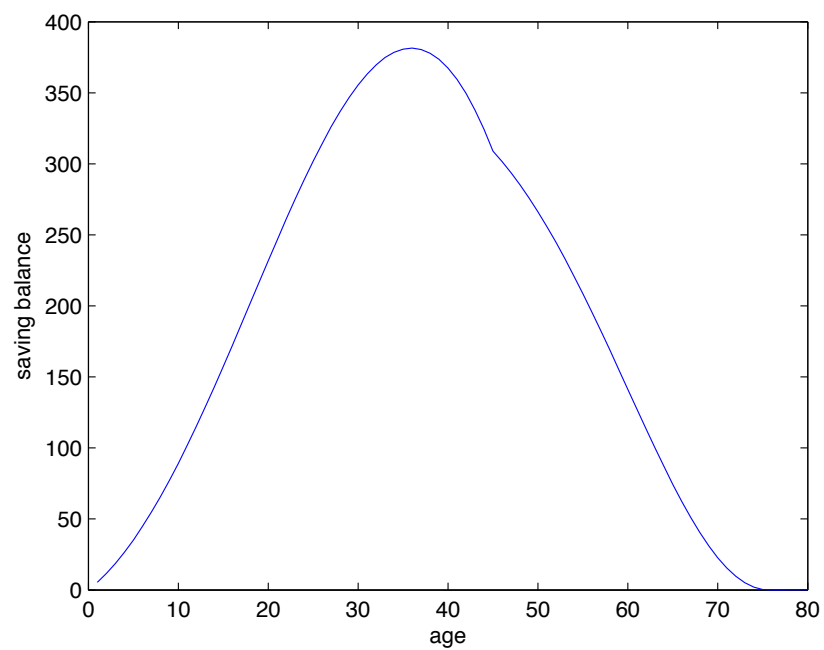


Figure A3–Balance of asset over the life cycle at the initial balanced growth path

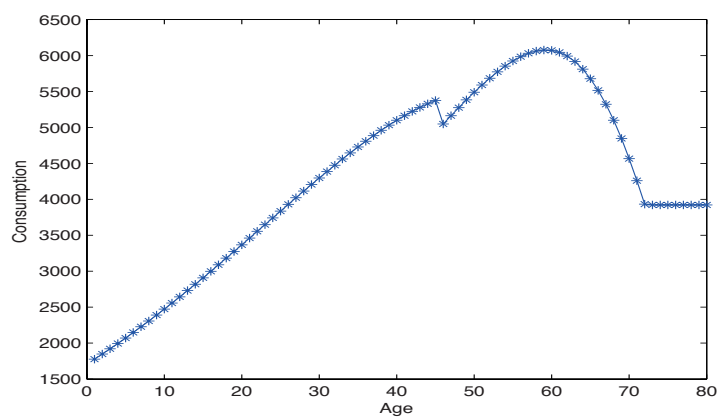


Figure A4–Life cycle consumption path at the initial balanced growth path.

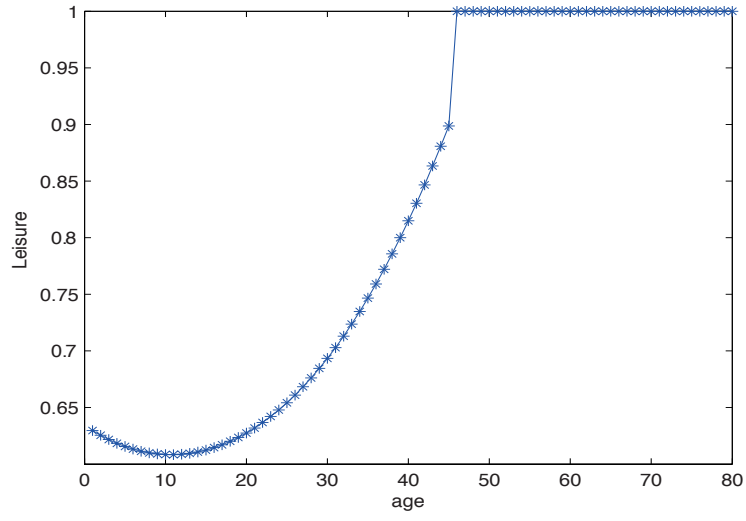


Figure A5—Life cycle consumption of leisure at the initial balanced growth path.

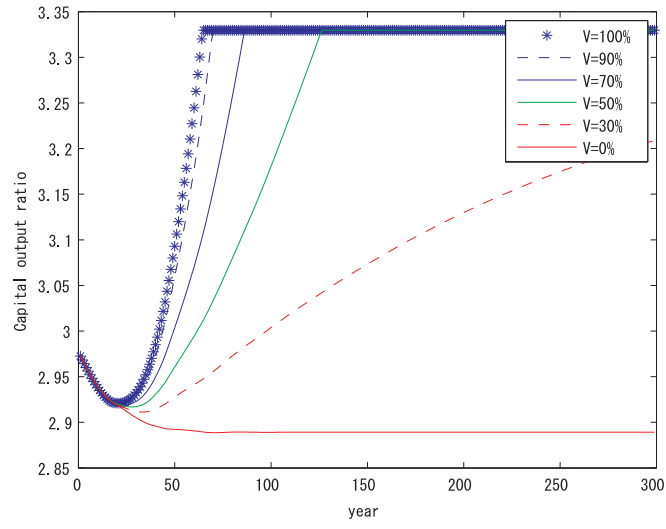


Figure A6—Capital to output ratio over time for different values of the share of the surplus used for government savings (V). In those simulations, the number of years needed for the PITN to reach the target level is set at 80. The target capital stock is the modified golden rule level. The intergenerational discount rate for the modified golden rule is set at 3%.

Table A1 Number of birth at each age for native and immigrnt in the simulation

age	number of birth of immigrant	number of birth of native
1	0.1934279	0.1743087
2	0.0520106	0.0474751
3	0.0550358	0.0505587
4	0.0569373	0.0527624
5	0.0578227	0.054141
6	0.0577998	0.0547487
7	0.0569765	0.0546402
8	0.0554604	0.0538698
9	0.0533594	0.052492
10	0.0507811	0.0505614
11	0.0478333	0.0481323
12	0.044624	0.0452592
13	0.0412605	0.0419967
14	0.0378511	0.0383992
15	0.0345031	0.034521
16	0.0313245	0.0304168
17	0.0284231	0.026141
18	0.0259064	0.0217481
19	0.0238824	0.0172925
20	0.0224589	0.0128286
21	0.0217435	0.0084112
22	0.0218439	0.0040944
23-80	0	0

Note

1. The calculaton is based on Figure 3. Let $TB(i,j)$ be the vertical axis of group j of Figure 3 where j is native or immigrant. Then, the number of births of age i of group j in the model is calcluated as follows. When $i=1$, the number of births of age i of group j in the model is $TB(20,j)/2$. When $2 \leq i \leq 22$, the number of births is $(TB(19+i,j)-TB(18+i,j))/2$.

Table A2
The effect of increasing the PITN for different inter-generational discount rates

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	inter-generation al discount rate for the modified golden rule	year reaching the modified golden rule	% increase of the capital stock per efficient unit labor at the	%change of publicly provided private goods per capita at the golden	% change of welfare of the cohort born at the golden rule	share of the PDV of increased publicly provided private goods in	share of the PDV of welfare gain of all natives and their descendants
1	80	0.0%	112	102.43%	35.86%	3.77%	12.23%	10.69%
2	80	0.5%	103	82.39%	35.07%	3.69%	15.39%	13.87%
3	80	1.0%	95	65.46%	33.04%	3.48%	18.10%	16.75%
4	80	1.5%	87	50.99%	30.14%	3.17%	20.26%	19.22%
5	80	2.0%	80	38.52%	26.63%	2.80%	21.80%	21.17%
6	80	2.5%	73	27.69%	22.71%	2.39%	22.64%	22.51%
7	80	3.0%	65	18.19%	18.51%	1.95%	22.71%	23.14%
8	80	3.5%	56	9.82%	14.12%	1.49%	21.86%	22.89%
9	80	4.0%	45	2.40%	9.64%	1.01%	19.80%	21.43%

Note

1. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.

Appendices B(The following appendices are for the purpose of refereeing.)

Appendix B1

Notice that in the programming problem, the objective function is concave and the constrained set is convex. Thus, if some allocation satisfies the first-order condition, it is also the solution of the programming problem. Now set up the Lagrangian function as follows:

$$\begin{aligned}
 L = & \frac{1}{1+\rho} [u^o(c_1^o) + v^o(g^o, q)] \\
 & + \sum_{t=1}^{\infty} \gamma_t \{ u^y(c_t^y, l_t) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho} [u^o(c_{t+1}^o) + v^o(g^o, g^{ind}) - u^*] \} \\
 & + \sum_{t=1}^{\infty} \lambda_t \{ F(R(\alpha^*)l_t, (s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \\
 & \quad - (c_t^o + g^o + g^{ind}) - R(\alpha^*) \times (c_t^y + s_t + a_t + g^y + g^{ind}) \}
 \end{aligned}$$

$$\text{where } a_0 = a^* \quad (67)$$

The first order conditions are:

$$\begin{aligned}
 c_1^o : \frac{1}{1+\rho} u^{o'}(c_1^o) &= \lambda_1; c_{t+1}^o : \gamma_t \frac{1}{1+\rho} u^{o'}(c_{t+1}^o) = \lambda_{t+1}; \\
 c_t^y : \gamma_t \frac{\partial u^y(c_t^y, l_t)}{\partial c_t^y} &= \lambda_t R(\alpha^*); l_t : \gamma_t \frac{\partial u^y(c_t^y, l_t)}{\partial l_t} = \lambda_t \frac{\partial F}{\partial L} R(\alpha^*) \\
 \gamma_t : u^y(c_t^y, l_t) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho} [u^o(c_{t+1}^o) + v^o(g^o, g^{ind}) - u^*] &= 0; \quad (68)
 \end{aligned}$$

$$\begin{aligned}
 s_t, a_t : \lambda_{t+1} \left\{ \frac{\partial F}{\partial K} + 1 - \delta \right\} &= \lambda_t R(\alpha^*) \\
 \lambda_t : F(R(\alpha^*), s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \\
 - (c_t^o + g^o + q) - R(\alpha^*)(c_t^y + s_t + a_t + g^o + q) &= 0 \quad (69)
 \end{aligned}$$

Those first order conditions imply that

$$\left(\frac{\partial u^y(c_t^y, l_t)}{\partial l_t} \right) / \left(\frac{\partial u^y(c_t^y, l_t)}{\partial c_t^y} \right) = \frac{\partial F}{\partial L}$$

$$\frac{\partial F}{\partial K} + 1 - \delta = \left(\frac{\partial u^y(c_t^y, l_t)}{\partial c_t^y} \right) / \left(\frac{1}{1 + \rho} u^{o'}(c_{t+1}^o) \right)$$

On the other hand, at the initial steady state, the initial steady state allocation, $(c^{y*}, c^{o*}, s^*, l^*, a^*)$ satisfy the following allocation:

$$- \left(\frac{\partial u^y(c^{y*}, l^*)}{\partial l} \right) / \left(\frac{\partial u^y(c^{y*}, l^*)}{\partial c^y} \right) = w^* = \frac{\partial F(R(\alpha^*)l^*, s^* + a^*)}{\partial L}$$

$$\frac{\partial F(R(\alpha^*)l^*, s^* + a^*)}{\partial K} + 1 - \delta = 1 + r^* = \left(\frac{\partial u^y(c^{y*}, l^*)}{\partial c_t^y} \right) / \left(\frac{1}{1 + \rho} u^{o'}(c^{o*}) \right)$$

$$u^y(c^{y*}, l^*) + v^y(g^y, g^{ind}) + \frac{1}{1 + \rho} [u^o(c^{o*}) + v^o(g^o, g^{ind})] = u^*$$

$$F(R(\alpha^*)l^*, s^* + a^*) + (1 - \delta)(s^* + a^*) = R(\alpha^*)(c^{y*} + s^* + a^* + g^o + g^{ind}) + (c^{o*} + g^o + g^{ind})$$

Now, we set $c_t^y, c_{t+1}^o, l_t, s_t, a_t, \lambda_t, \gamma_t$ as follows

$$c_t^o = c^{o*}; c_t^y = c^{y*}; s_t = s^*; l_t = l^*; a_t = a^*; \lambda_1 = \frac{1}{1 + \rho} u^{o'}(c^{o*})$$

$$\lambda_{t+1} = \lambda_t \frac{R(\alpha^*)}{1 + r^*}; \gamma_t \frac{1}{1 + \rho} u^{o'}(c^{o*}) = \lambda_{t+1}$$

If $c_t^y, c_{t+1}^o, s_t, a_t, \lambda_t, \gamma_t$ are set in this way, it clearly satisfies the first-order conditions of the programming problem. Thus, the initial steady state allocation is Pareto-efficient. Q.E.D.

Appendix B2

Using the definitions of τ_{wt} and τ_{rt} , we have

$$w_t \tau_{wt} = w_t - (1 - \tau_w^*) w^*$$

$$r_t \tau_{rt} = r_t - (1 - \tau_r^*) r^*$$

Then, the government budget surplus at period 1 is

$$\begin{aligned}
SP_1 = & (w_1 - (1 - \tau_w^*)w^*)l^* \sum_{j=n,m} N_1^j + (r_1 - (1 - \tau_r^*)r^*)s^* \sum_{j=n,m} N_0^j \\
& - (b^* + g^o + g^{ind}) \sum_{j=n,m} N_0^j - (g^y + g^{ind} + a^*) \sum_{j=n,m} N_1^j + a^*(1 + r_1) \sum_{j=n,m} N_0^j \quad (70)
\end{aligned}$$

Note that $N_1^m = N_1^n \tilde{\alpha}$ where $\tilde{\alpha} > \alpha^*$ and that N_1^n is pre-determined where $N_1^n = N_1^0(1 + \pi_n) + N_0^n(1 + \pi_m)$ and $N_0^m = N_0^n \alpha^*$.

$$\begin{aligned}
SP_1 = & w_1 l^* \sum_{j=n,m} N_1^j - w^*(1 - \tau_w^*)l^* \sum_{j=n,m} N_1^j \\
& + r_1 s^* N_0^n(1 + \alpha^*) - r^* s^*(1 - \tau_r^*)N_0^n(1 + \alpha^*) - N_0^n(1 + \alpha^*)(b^* + g^o + g^{ind}) \\
& - (g^y + g^{ind} + a^*)\{N_1^n(1 + \alpha^*) + N_1^n(\tilde{\alpha} - \alpha^*)\} + N_0^n(1 + \alpha^*)a^*(1 + r_1) \quad (71)
\end{aligned}$$

Also notice that $\sum_{j=n,m} N_1^j = N_1^n(1 + \tilde{\alpha}) = N_1^n(1 + \alpha^* + \tilde{\alpha} - \alpha^*)$ and $\sum_{j=n,m} N_0^j = N_0^n(1 + \alpha^*)$. Thus, SP_1 becomes

$$\begin{aligned}
SP_1 = & w_1 l^* \sum_{j=n,m} N_1^j - w^*(1 - \tau_w^*)l^* \{N_1^n(1 + \alpha^*) + N_1^n(\tilde{\alpha} - \alpha^*)\} \\
& + r_1 s^* N_0^n(1 + \alpha^*) - r^* s^*(1 - \tau_r^*)N_0^n(1 + \alpha^*) - N_0^n(1 + \alpha^*)(b^* + g^o + g^{ind}) \\
& - (g^y + g^{ind} + a^*)\{N_1^n(1 + \alpha^*) + N_1^n(\tilde{\alpha} - \alpha^*)\} + N_0^n(1 + \alpha^*)a^*(1 + r_1) \quad (72)
\end{aligned}$$

At the steady state, as for the government budget constraint, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind} - a^*)N_1^n(1 + \alpha^*) + (\tau_r^* r^* s^* - b^* - g^o - g^{ind})N_0^n(1 + \alpha^*) + N_0^n(1 + \alpha^*)a^*(1 + r^*) = 0 \quad (73)$$

By using the government budget constraint at the initial steady state, we can

rewrite SP_1 as follows:

$$\begin{aligned}
SP_1 = & w_1 l^* \sum_{j=n,m} N_1^j - w^* l_1^* N_1^n (1 + \alpha^*) - w^* l^* (1 - \tau_w^*) N_1^n (\tilde{\alpha} - \alpha^*) - N_0^n (1 + \alpha^*) a^* (1 + r^*) \\
& + r_1 s^* N_0^n (1 + \alpha^*) - r^* s^* N_0^n (1 + \alpha^*) - (g^y + g^{ind} + a^*) N_1^n (\tilde{\alpha} - \alpha^*) + N_0^n (1 + \alpha^*) a^* (1 + r_1)
\end{aligned}$$

$N_0^n (1 + \alpha^*) a^*$ can be canceled out in the above equation. Thus we have

$$\begin{aligned}
SP_1 = & w_1 l^* \sum_{j=n,m} N_1^j - w^* l_1^* N_1^n (1 + \alpha^*) - w^* l^* (1 - \tau_w^*) N_1^n (\tilde{\alpha} - \alpha^*) \\
& + r_1 s^* N_0^n (1 + \alpha^*) - r^* s^* N_0^n (1 + \alpha^*) - (g^y + g^{ind} + a^*) N_1^n (\tilde{\alpha} - \alpha^*) + N_0^n (1 + \alpha^*) a^* (r_1 - r^*)
\end{aligned}$$

From the homogeneity of the production function and Euler's theorem, we have

$$w_1 l^* \sum_{j=n,m} N_1^j + r_1 (s^* + a^*) N_0^n (1 + \alpha^*) = F(l^* \sum_{j=n,m} N_1^j, (s^* + a^*) N_0^n (1 + \alpha^*)) - \delta (s^* + a^*) N_0^n (1 + \alpha^*)$$

At the initial steady-state, we also have

$$w^* l^* N_1^n (1 + \alpha^*) + r^* (s^* + a^*) N_0^n (1 + \alpha^*) = F(l^* N_1^n (1 + \alpha^*), (s^* + a^*) N_0^n (1 + \alpha^*)) - \delta (s^* + a^*) N_0^n (1 + \alpha^*)$$

Thus, SP_1 becomes

$$\begin{aligned}
SP_1 = & F(l^* \sum_{j=n,m} N_1^j, (s^* + a^*) N_0^n (1 + \alpha^*)) - \delta (s^* + a^*) N_0^n (1 + \alpha^*) \\
& - \{F(l_1^* N_1^n (1 + \alpha^*), (s^* + a^*) N_0^n (1 + \alpha^*)) \\
& - \delta (s^* + a^*) N_0^n (1 + \alpha^*)\} - w^* l^* (1 - \tau_w^*) N_1^n (\tilde{\alpha} - \alpha^*) - N_1^n (g^y + g^{ind} + a^*) (\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{74}$$

Note that $N_1^m = \tilde{\alpha}N_1^n$. Thus,

$$\begin{aligned}
SP_1 &= F(l^*(N_1^n(1 + \tilde{\alpha}), (s^* + a^*)N_0^n(1 + \alpha^*)) - F(l^*\{N_1^n(1 + \alpha^*)\}, (s^* + a^*)N_0^n(1 + \alpha^*)) \\
&\quad - \{w^*l^*(1 - \tau_w^*) - (g^y + g^{ind} + a^*)\}N_1^n(\tilde{\alpha} - \alpha^*)) \\
&= \int_{1+\alpha^*}^{1+\tilde{\alpha}} [F_L(N_1^n l^* z, (s^* + a^*)N_0^n(1 + \alpha^*))N_1^n l^* - w^*(1 - \tau_w)N_1^n l^* - (g^y + g^{ind} + a^*)N_1^n] dz \\
&= N_1^n \int_{1+\alpha^*}^{1+\tilde{\alpha}} F_L(N_1^n l^* z, (s^* + a^*)N_0^n(1 + \alpha^*))l^* - w^*l^*(1 - \tau_w) - (g^y + g^{ind} + a^*)] dz
\end{aligned} \tag{75}$$

Note that $w^*l^*(1 - t_w) = c^{y*} + s^*$. Thus, we have

$$= N_1^n \int_{1+\alpha^*}^{1+\tilde{\alpha}} F_L(N_1^n l^* z, (s^* + a^*)N_0^n(1 + \alpha^*))l^* - c^{y*} - s^* - g^y - g^{ind} - a^*)] dz \tag{76}$$

Note that $N_1^n = R(\alpha^*)N_0^n$ and using the homogeneity of F_L , we have

$$SP_1 = N_1 \int_{1+\alpha^*}^{1+\tilde{\alpha}} F_L(R(\alpha^*)l^* z, (s^* + a^*)(1 + \alpha^*))l^* - c^{y*} - s^* - g^y - g^{ind} - a^*)] dz$$

Appendix B3

Note that $\sum_{j=n,m} N_2^j = N_2^n(1 + \tilde{\alpha})$ and $\sum_{j=n,m} N_1^j = N_1^n(1 + \tilde{\alpha})$. Thus, SP_2 becomes as follows:

$$\begin{aligned}
SP_2 &= w_2\tau_{w2}l^*N_2^n(1 + \tilde{\alpha}) + r_2\tau_{r2}s^*N_1^n(1 + \tilde{\alpha}) - N_1^n(1 + \tilde{\alpha}) \times (b^* + g^o + g^{ind}) \\
&\quad - N_2^n(1 + \tilde{\alpha}) \times (g^y + g^{ind} + a^*) + (1 + r_2)a_1N_1^n(1 + \tilde{\alpha})
\end{aligned} \tag{77}$$

Using the definitions of τ_{w2} and τ_{r2} , we have

$$\begin{aligned}
SP_2 &= w_2 l^* N_2^n (1 + \tilde{\alpha}) - w^* (1 - \tau_w^*) l^* N_2^n (1 + \tilde{\alpha}) \\
&\quad + r_2 s^* N_1^n (1 + \tilde{\alpha}) - r^* (1 - \tau_r^*) s^* N_1^n (1 + \tilde{\alpha}) - N_1^n (1 + \tilde{\alpha}) (b + g^o + g^{ind}) \\
&\quad - N_2^n (1 + \tilde{\alpha}) (g^y + g^{ind} + a^*) + (1 + r_2) a_1 N_1^n (1 + \tilde{\alpha})
\end{aligned} \tag{78}$$

By changing the order in the above equation, SP_2 becomes

$$\begin{aligned}
SP_2 &= w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 s^* N_1^n (1 + \tilde{\alpha}) \\
&\quad - w^* l^* N_2^n (1 + \tilde{\alpha}) - r^* s^* N_1^n (1 + \tilde{\alpha}) \\
&\quad + \tau_w^* w^* l^* N_2^n (1 + \tilde{\alpha}) + \tau_r^* r^* s^* N_1^n (1 + \tilde{\alpha}) - N_1^n (1 + \tilde{\alpha}) (b + g^o + g^{ind}) \\
&\quad - N_2^n (1 + \tilde{\alpha}) (g^y + g^{ind} + a^*) + (1 + r_2) a_1 N_1^n (1 + \tilde{\alpha}) \\
&= w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 s^* N_1^n (1 + \tilde{\alpha}) + (1 + r_2) a_1 N_1^n (1 + \tilde{\alpha}) \\
&\quad - w^* l^* N_2^n (1 + \tilde{\alpha}) - r^* s^* N_1^n (1 + \tilde{\alpha}) \\
&\quad + (1 + \tilde{\alpha}) \{ \tau_w^* w^* l^* N_2^n + \tau_r^* r^* s^* N_1^n - N_1^n (b + g^o + g^{ind}) - N_2^n (g^y + g^{ind} + a^*) \}
\end{aligned} \tag{79}$$

Now we calculate $\tau_w^* w^* l^* N_2^n + \tau_r^* r^* s^* N_1^n - N_1^n (b + g^o + g^{ind}) - N_2^n (g^y + g^{ind} + a^*)$. Note

that $N_2^n = N_1^n ((1 + \pi_m) \tilde{\alpha} + 1 + \pi_n) = N_1^n ((1 + \pi_m) \alpha^* + 1 + \pi_n + (1 + \pi_m) \tilde{\alpha} - (1 + \pi_m) \alpha^*)$.

Thus,

$$\begin{aligned}
&\tau_w^* w^* l^* N_2^n + \tau_r^* r^* s^* N_1^n - N_1^n (b + g^o + g^{ind}) - (g^y + g^{ind} + a^*) N_2^n \\
&= \tau_w^* w^* l^* N_1^n (R(\alpha^*) + (1 + \pi_m) (\tilde{\alpha} - \alpha^*)) \\
&\quad + \tau_r^* r^* s^* N_1^n - N_1^n (b + g^o + g^{ind}) \\
&\quad - (g^y + g^{ind} + a^*) N_1^n (R(\alpha^*) + (1 + \pi_m) (\tilde{\alpha} - \alpha^*))
\end{aligned} \tag{80}$$

At the initial steady state, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind} - a^*)R(\alpha^*)N_0^n(1 + \alpha^*) + (\tau_r^* r^* s^* + (1 + r^*)a^* - b^* - g^o)N_0^n(1 + \alpha^*) = 0 \quad (81)$$

By dividing by $N_0^n(1 + \alpha^*)$, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind} - a^*)R(\alpha^*) + (\tau_r^* r^* s^* + (1 + r^*)a^* - b^* - g^o) = 0 \quad (82)$$

Thus, (80) becomes

$$= \tau_w^* w^* l^* N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (g^y + g^{ind} + a^*)N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (1 + r^*)a^* N_1^n \quad (83)$$

Therefore, SP_2 becomes

$$\begin{aligned} SP_2 &= w_2 l^* N_2^n(1 + \tilde{\alpha}) + r_2 s^* N_1^n(1 + \tilde{\alpha}) + (1 + r_2)a_1 N_1^n(1 + \tilde{\alpha}) \\ &\quad - w^* l^* N_2^n(1 + \tilde{\alpha}) - r^* s^* N_1^n(1 + \tilde{\alpha}) \\ &\quad + (1 + \tilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\ &\quad - (g^y + g^{ind} + a^*)N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (1 + r^*)a^* N_1^n\} \end{aligned} \quad (84)$$

Now, we decompose $w^* l^* N_2^n(1 + \tilde{\alpha})$ in the second line in the above equation. Since $N_2^n = N_1^n R(\tilde{\alpha}) = N_1^n(R(\alpha^*) + (1 + \pi_m)(\tilde{\alpha} - \alpha^*))$, SP_2 becomes

$$\begin{aligned} SP_2 &= w_2 l^* N_2^n(1 + \tilde{\alpha}) + r_2 s^* N_1^n(1 + \tilde{\alpha}) + (1 + r_2)a_1 N_1^n(1 + \tilde{\alpha}) \\ &\quad - (1 + \tilde{\alpha})w^* l^* N_1^n\{R(\alpha^*) + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)\} - r^* s^* N_1^n(1 + \tilde{\alpha}) \\ &\quad + (1 + \tilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (g^y + g^{ind} + a^*)N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (1 + r^*)a^* N_1^n\} \end{aligned}$$

Re-arranging the second line in the above equation, we have

$$\begin{aligned} SP_2 &= w_2 l^* N_2^n(1 + \tilde{\alpha}) + r_2 s^* N_1^n(1 + \tilde{\alpha}) + (1 + r_2)a_1 N_1^n(1 + \tilde{\alpha}) \\ &\quad - (1 + \tilde{\alpha})w^* l^* N_1^n R(\alpha^*) - (1 + \tilde{\alpha})w^* l^* N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - r^* s^* N_1^n(1 + \tilde{\alpha}) \\ &\quad + (1 + \tilde{\alpha})\{\tau_w^* w^* l^* N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (g^y + g^{ind} + a^*)N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (1 + r^*)a^* N_1^n\} \end{aligned}$$

Next, we re-arrange $r_2 s^* N_1^n (1 + \tilde{\alpha}) + (1 + r_2) a_1 N_1^n (1 + \tilde{\alpha})$ and $(1 + r^*) a^* N_1^n$. Then, we have

$$\begin{aligned} SP_2 &= w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 (s^* + a_1) N_1^n (1 + \tilde{\alpha}) + a_1 N_1^n (1 + \tilde{\alpha}) \\ &\quad - (1 + \tilde{\alpha}) w^* l^* N_1^n R(\alpha^*) - r^* (s^* + a^*) N_1^n (1 + \tilde{\alpha}) - (1 + \tilde{\alpha}) w^* l^* N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\ &\quad + (1 + \tilde{\alpha}) \{ \tau_w^* w^* l^* N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind} + a^*) N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - a^* N_1^n \} \end{aligned}$$

Using the homogeneity of the production function, we have

$$\begin{aligned} w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 (s^* + a_1) N_1^n (1 + \tilde{\alpha}) &= F(l^* N_2^n (1 + \tilde{\alpha}), (s^* + a_1) N_1^n (1 + \tilde{\alpha})) - \delta (s^* + a_1) N_1^n (1 + \tilde{\alpha}) \\ &\text{and} \end{aligned}$$

$$\begin{aligned} &w^* l^* (1 + \alpha^*) N_1^n R(\alpha^*) + r^* (s^* + a^*) N_1^n (1 + \alpha^*) \\ &= F(l^* (1 + \alpha^*) N_1^n R(\alpha^*), (s^* + a^*) N_1^n (1 + \alpha^*)) \\ &\quad - \delta (s^* + a^*) N_1^n (1 + \alpha^*) \end{aligned}$$

Thus, SP_2 becomes

$$\begin{aligned} SP_2 &= F(l^* N_2^n (1 + \tilde{\alpha}), (s^* + a_1) N_1^n (1 + \tilde{\alpha})) - \delta (s^* + a_1) N_1^n (1 + \tilde{\alpha}) + a_1 N_1^n (1 + \tilde{\alpha}) \\ &\quad - \frac{1 + \tilde{\alpha}}{1 + \alpha^*} \{ F(l^* (1 + \alpha^*) N_1^n R(\alpha^*), (s^* + a^*) N_1^n (1 + \alpha^*)) - \delta (s^* + a^*) N_1^n (1 + \alpha^*) \} \\ &\quad - (1 + \tilde{\alpha}) w^* l^* N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\ &\quad + (1 + \tilde{\alpha}) \{ \tau_w^* w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) N_1^n - (g^y + g^{ind} + a^*) N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - a^* N_1^n \} \end{aligned} \tag{85}$$

Combining the third line and fourth line, we have

$$\begin{aligned}
SP_2 = & F(l^*N_2^n(1+\tilde{\alpha}), (s^*+a_1)N_1^n(1+\tilde{\alpha})) - \delta(s^*+a_1)N_1^n(1+\tilde{\alpha}) + a_1N_1^n(1+\tilde{\alpha}) \\
& - F(l^*(1+\tilde{\alpha})N_1^nR(\alpha^*), (s^*+a^*)N_1^n(1+\tilde{\alpha})) + \delta(s^*+a^*)N_1^n(1+\tilde{\alpha}) \\
& + (1+\tilde{\alpha})N_1^n\{-(1-\tau_w)w^*l^*(1+\pi_m)(\tilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)(1+\pi_m)(\tilde{\alpha}-\alpha^*) - a^*N_1^n\}
\end{aligned} \tag{86}$$

Notice that $\delta s^*N_1^n(1+\tilde{\alpha})$ is canceled out in the above equation. Rearranging the term $(a_1-a^*)N_1^n(1+\tilde{\alpha})$, we have

$$\begin{aligned}
SP_2 = & F(l^*N_2^n(1+\tilde{\alpha}), (s^*+a_1)N_1^n(1+\tilde{\alpha})) + (a_1-a^*)N_1^n(1+\tilde{\alpha}) - \delta(a_1-a^*)N_1^n(1+\tilde{\alpha}) \\
& - F(l^*(1+\tilde{\alpha})N_1^nR(\alpha^*), (s^*+a^*)N_1^n(1+\tilde{\alpha})) \\
& + (1+\tilde{\alpha})N_1^n\{-(1-\tau_w)w^*l^*(1+\pi_m)(\tilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)(1+\pi_m)(\tilde{\alpha}-\alpha^*)\}
\end{aligned}$$

Subtracting and adding $F(l^*N_2^n(1+\tilde{\alpha}), (s^*+a^*)N_1^n(1+\tilde{\alpha}))$, SP_2 becomes

$$\begin{aligned}
SP_2 = & F(l^*N_2^n(1+\tilde{\alpha}), (s^*+a_1)N_1^n(1+\tilde{\alpha})) + (1-\delta)(a_1-a^*)N_1^n(1+\tilde{\alpha}) \\
& - F(l^*N_2^n(1+\tilde{\alpha}), (s^*+a^*)N_1^n(1+\tilde{\alpha})) \\
& + F(l^*N_2^n(1+\tilde{\alpha}), (s^*+a^*)N_1^n(1+\tilde{\alpha})) - F(l^*(1+\tilde{\alpha})N_1^nR(\alpha^*), (s^*+a^*)N_1^n(1+\tilde{\alpha})) \\
& + (1+\tilde{\alpha})N_1^n\{-(1-\tau_w)w^*l^*(1+\pi_m)(\tilde{\alpha}-\alpha^*) - (g^y+g^{ind}+a^*)(1+\pi_m)(\tilde{\alpha}-\alpha^*)\}
\end{aligned} \tag{87}$$

For the first and the second line in the above equation, it can be re-written as

$$\begin{aligned}
&= F(l^* N_1^n R(\tilde{\alpha})((1 + \tilde{\alpha}), (s^* + a_1) N_1^n(1 + \tilde{\alpha})) + (1 - \delta)(a_1 - a^*) N_1^n(1 + \tilde{\alpha}) \\
&\quad - F(l^{**} N_1^n R(\tilde{\alpha})(1 + \tilde{\alpha}), (s^* + a^*) N_1^n(1 + \tilde{\alpha})) \\
&= N_1^n(1 + \tilde{\alpha}) \{ F(l^* R(\tilde{\alpha}), s^* + a_1) + (1 - \delta)(a_1 - a^*) \\
&\quad - F(l^{**}(1 + \tilde{\alpha}), s^* + a^*) \} \\
&= N_1^n(1 + \tilde{\alpha}) \int_{s^* + a^*}^{s^* + a_1} [F_K(l^* R(\tilde{\alpha}), z) + (1 - \delta)] dz
\end{aligned}$$

Next, we focus on the third and fourth lines of (87) . Note that $N_2^n = N_1^n R(\tilde{\alpha})$. Thus, the third and fourth line of (87) can be re-written as

$$\begin{aligned}
&\int_{\alpha^*}^{\tilde{\alpha}} [F_L(l^*(1 + \tilde{\alpha}) N_1^n R(z), (s^* + a^*) N_1^n(1 + \tilde{\alpha})) l^*(1 + \tilde{\alpha}) N_1^n(1 + \pi_m) \\
&\quad - (1 + \tilde{\alpha}) N_1^n(1 + \pi_m) \{ (1 - \tau_w) w^* l^* + g^y + g^{ind} + a^* \}] dz \\
&= (1 + \tilde{\alpha}) N_1^n(1 + \pi_m) \int_{\alpha^*}^{\tilde{\alpha}} [F_L(l^*(1 + \tilde{\alpha}) N_1^n R(z), (s^* + a^*) N_1^n(1 + \tilde{\alpha})) l^* \\
&\quad - ((1 - \tau_w) w^* l^* + g^y + g^{ind} + a^*)] dz
\end{aligned}$$

Since the F_L is homogenous degree of zero, the above equation becomes

$$\begin{aligned}
&= (1 + \tilde{\alpha}) N_1^n(1 + \pi_m) \int_{\alpha^*}^{\tilde{\alpha}} [F_L(l^* R(z), (s^* + a^*)) l^* \\
&\quad - \{ (1 - \tau_w) w^* l^* + g^y + g^{ind} + a^* \}] dz
\end{aligned}$$

Note that $(1 - \tau_w) w^* l^* = c^{y^*} + s^*$ and $R'(\alpha) = 1 + \pi_m$. Therefore, SP_2 becomes as follows:

$$\begin{aligned}
SP_2 = & N_1^n(1 + \tilde{\alpha}) \int_{s^*+a^*}^{s^*+a_1} [F_K(l^*R(\tilde{\alpha}), z) + 1 - \delta] dz \\
& (1 + \tilde{\alpha})N_1^n \int_{\alpha^*}^{\tilde{\alpha}} R'(z) [F_L(l^*R(z), s^* + a^*)l^* \\
& - \{c^{y^*} + s^* + g^y + g^{ind} + a^*\}] dz
\end{aligned}$$

Appendix B4

To save space, I will show that $SP_t > 0$ for $t = 2, 3, \dots$. For $t = 1$, the same proof is applied as in the appendix B2.

We assume the same tax adjustment as in the preceding subsection.

$$(1 - \tau_{wt})w_t = (1 - \tau_w^*)w^* \text{ and } (1 - \tau_{rt})r_t = (1 - \tau_r^*)r^* \quad (88)$$

With this tax adjustment, labor supply and saving of each individual is the same as at the initial steady state. As in the proof in the appendix B2 and B3, I assume that the government will save at least the same amount of the government saving per each young individual as at the initial steady state. Note that

$$\begin{aligned}
SP_t = & w_t \tau_{wt} l^{n*} N_t^n + \phi^m w_t \tau_{wt} l^{m*} N_t^m + \tau_{rt} r_t s^{n*} N_{t-1}^n + \tau_{rt} r_t s^{m*} N_{t-1}^m \\
& - N_t^n (g^{y,n} + g^{ind,m} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m}) \\
& + (1 + r_t) a_{t-1} (N_{t-1}^n + N_{t-1}^m)
\end{aligned} \quad (89)$$

Using the definition of τ_{wt} and τ_{rt} , we have

$$w_t \tau_{wt} = w_t - (1 - \tau_w^*)w^* \quad (90)$$

$$r_t \tau_{rt} = r_{t+1} - (1 - \tau_r^*)r^* \quad (91)$$

Thus, SP_t becomes

$$\begin{aligned}
SP_t = & w_t l^{n*} N_t^n - w^* l^{n*} N_t^n + \tau_w^* w^* l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m - \phi^m w^* l^{m*} N_t^m + \phi^m \tau_w^* w^* l^{m*} N_t^m \\
& + r_t s^{n*} N_{t-1}^n - r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{n*} N_{t-1}^n + \\
& + r_t s^{m*} N_{t-1}^m - r^* s^{m*} N_{t-1}^m + \tau_r^* r^* s^{m*} N_{t-1}^m \\
& - N_t^n \times (g^{y,n} + g^{ind,n} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m}) \\
& + (1 + r_t) a_{t-1} (N_{t-1}^n + N_{t-1}^m)
\end{aligned} \tag{92}$$

By changing the order of the above equation, we have

$$\begin{aligned}
SP_t = & w_t l^{n*} N_t^n - w^* l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m - \phi^m w^* l^{m*} N_t^m \\
& + r_t s^{n*} N_{t-1}^n - r^* s^{n*} N_{t-1}^n + r_t s^{m*} N_{t-1}^m - r^* s^{m*} N_{t-1}^m \\
& + (1 + r_t) a_{t-1} (N_{t-1}^n + N_{t-1}^m) \\
& + \tau_w^* w^* l^{n*} N_t^n + \phi^m \tau_w^* w^* l^{m*} N_t^m \\
& + \tau_r^* r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{m*} N_{t-1}^m \\
& - N_t^n \times (g^{y,n} + g^{ind,n} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned}$$

Now we need to calculate the fourth line to seventh line:

$$\begin{aligned}
& \tau_w^* w^* l^{n*} N_t^n + \phi^m \tau_w^* w^* l^{m*} N_t^m + \tau_r^* r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{m*} N_{t-1}^m \\
& - N_t^n (g^{y,n} + g^{ind,n} + a^*) - N_t^m (g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^m (b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned} \tag{93}$$

Note that $N_t^n = N_{t-1}^n R(\tilde{\alpha})$ and $N_t^m = N_{t-1}^m \tilde{\alpha}$. Thus, the above equation becomes as

follows:

$$\begin{aligned}
& \tau_w^* w^* l^{n*} N_{t-1}^n R(\tilde{\alpha}) + \phi^m \tau_w^* w^* l^{n*} N_{t-1}^n J(\tilde{\alpha}) \\
& + \tau_r^* r^* s^{n*} N_{t-1}^n + \tau_r^* r^* s^{m*} N_{t-1}^n \tilde{\alpha} \\
& - N_{t-1}^n R(\tilde{\alpha})(g^{y,n} + g^{ind,n} + a^*) \\
& - N_{t-1}^n J(\tilde{\alpha})(g^{y,m} + g^{ind,m} + a^*) \\
& - N_{t-1}^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_{t-1}^n \tilde{\alpha} (b^{m*} + g^{o,m} + g^{ind,m}) \tag{94}
\end{aligned}$$

On the other hand, at the initial steady state, $N_t^n = N_{t-1}^n R(\alpha^*)$ and $N_t^m = N_{t-1}^m \alpha^*$. Thus, the government budget constraint at the initial steady state implies

$$\begin{aligned}
& \tau_w^* w^* l^{n*} R(\alpha^*) + \phi^m \tau_w^* w^* l^{m*} J(\alpha^*) \\
& + \tau_r^* r^* s^{n*} + \tau_r^* r^* s^{m*} \alpha^* \\
& - R(\alpha^*)(g^{y,n} + g^{ind,n} + a^*) - J(\alpha^*)(g^{y,m} + g^{ind,m} + a^*) \\
& - (b^{n*} + g^{o,n} + g^{ind,n}) - \alpha^* (b^{m*} + g^{o,m} + g^{ind,m}) + (1 + r^*) a^* (1 + \alpha^*) = 0 \tag{95}
\end{aligned}$$

Thus, (94) becomes

$$\begin{aligned}
& \tau_w^* w^* l^{n*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) (1 + \pi_m) \\
& + \phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\tilde{\alpha}) - J(\alpha^*)) \\
& + \tau_r^* r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*) (\tilde{\alpha} - \alpha^*) (1 + \pi_m) \\
& - N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*) (J(\tilde{\alpha}) - J(\alpha^*)) \\
& - N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*) \\
& - (1 + r^*) a^* N_{t-1}^n (1 + \alpha^*) \tag{96}
\end{aligned}$$

Therefore, SP_t becomes

$$\begin{aligned}
SP_t &= w_t l^{n*} N_t^n - w^* l^{n*} N_t^n \\
&+ \phi^m w_t l^{m*} N_t^m - \phi^m w^* l^{m*} N_t^m \\
&+ r_t s^{n*} N_{t-1}^n - r^* s^{n*} N_{t-1}^n \\
&+ r_t s^{m*} N_{t-1}^m - r^* s^{m*} N_{t-1}^m \\
&+ (1 + r_t) a_{t-1} (N_{t-1}^n + N_{t-1}^m) \\
&+ \tau_w^* w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
&+ \phi^m \tau_w^* w^* l^{m*} N_{t-1}^m \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&+ \tau_r^* r^* s^{m*} N_{t-1}^m (\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*) (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*) (J(\tilde{\alpha}) - J(\alpha^*)) \\
&- N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*) \\
&- (1 + r^*) a^* N_{t-1}^n (1 + \alpha^*) \\
&= w_t l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m \\
&+ r_t s^{n*} N_{t-1}^n + r_t s^{m*} N_{t-1}^m + r_t a_{t-1} (N_{t-1}^n + N_{t-1}^m) \\
&+ a_{t-1} (N_{t-1}^n + N_{t-1}^m) \\
&- \{w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m + r^* s^{n*} N_{t-1}^n + r^* s^{m*} N_{t-1}^m \\
&+ r^* a^* N_{t-1}^n (1 + \alpha^*)\} \\
&- a^* N_{t-1}^n (1 + \alpha^*) \\
&+ \tau_w^* w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
&+ \phi^m \tau_w^* w^* l^{m*} N_{t-1}^m \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&+ \tau_r^* r^* s^{m*} N_{t-1}^m (\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n (g^{y,n} + g^{ind,n} + a^*) (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*) (J(\tilde{\alpha}) - J(\alpha^*))
\end{aligned}$$

We add and subtract $\delta(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m))$ to and from SP_t . We also subtract and $\delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^*N_{t-1}^n + a^*N_{t-1}^n(1 + \alpha^*))$ to and from SP_t . Then, we have

$$\begin{aligned}
SP_t &= w_t l^{n*} N_t^n + \phi^m w_t l^{m*} N_t^m \\
&+ (r_t + \delta)(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) + a_{t-1}(N_{t-1}^n + N_{t-1}^m) \\
&- \delta(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
&- \{w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m \\
&+ r^* s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^* N_{t-1}^n(1 + \alpha^*)\} \\
&- \delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^*N_{t-1}^n + a^*(N_{t-1}^n(1 + \alpha^*))) \\
&+ \delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^*N_{t-1}^n + a^*N_{t-1}^n(1 + \alpha^*)) \\
&+ \tau_w^* w^* l^{n*} N_{t-1}^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&+ \phi^m \tau_w^* w^* l^{m*} N_{t-1}^n \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&+ \tau_r^* r^* s^{m*} N_{t-1}^n(\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n(g^{y,n} + g^{ind,n} + a^*)(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&- N_{t-1}^n(g^{y,m} + g^{ind,m} + a^*)\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&- N_{t-1}^n(b^{m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*) \\
&- a^* N_{t-1}^n(1 + \alpha^*)
\end{aligned} \tag{97}$$

Note that the first three line of the above equation becomes

$$\begin{aligned}
&F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
&+ (1 - \delta)a_{t-1}(N_{t-1}^n + N_{t-1}^m) - \delta(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m)
\end{aligned}$$

Next, we focus on $w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m + r^*(s^{n*}N_{t-1}^n + s^{m*}N_{t-1}^m + r^*a^*(N_{t-1}^n + \alpha^*N_{t-1}^n)) + \delta(s^{n*}N_{t-1}^n + s^{m*}\alpha^*N_{t-1}^n + a^*N_{t-1}^n(1 + \alpha^*))$. Note that $N_t^n = R(\tilde{\alpha})N$, $N_t^m =$

$N_{t-1}^n J(\tilde{\alpha}), N_{t-1}^m = N_{t-1}^n \tilde{\alpha}$. Thus, we have

$$\begin{aligned}
& w^* l^{n*} N_t^n + \phi^m w^* l^{m*} N_t^m + r^* (s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^* N_{t-1}^n (1 + \alpha^*)) \\
& + \delta (s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n + a^* N_{t-1}^n (1 + \alpha^*)) \\
& = w^* l^{n*} N_{t-1}^n R(\tilde{\alpha}) + \phi^m w^* l^{m*} N_{t-1}^n J(\tilde{\alpha}) \\
& + r^* s^{n*} N_{t-1}^n + r^* s^{m*} N_{t-1}^n \tilde{\alpha} + r^* a^* N_{t-1}^n (1 + \alpha^*) \\
& + \delta (s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n + a^* N_{t-1}^n (1 + \alpha^*)) \\
& = w^* l^{n*} N_{t-1}^n \{R(\alpha^*) + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)\} \\
& + \phi^m w^* l^{m*} N_{t-1}^n l^{m*} (J(\alpha^*) + J(\tilde{\alpha}) - J(\alpha^*)) \\
& + r^* s^{n*} N_{t-1}^n + r^* s^{m*} N_{t-1}^n (\alpha^* + \tilde{\alpha} - \alpha^*) + r^* a^* N_{t-1}^n (1 + \alpha^*) \\
& + \delta (s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n + r^* a^* N_{t-1}^n (1 + \alpha^*)) \\
& = w^* N_{t-1}^n [R(\alpha^*) l^{n*} + \phi^m J(\alpha^*) l^m] \\
& + (r^* + \delta) [s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^* N_{t-1}^n (1 + \alpha^*)] \\
& + w^* l^{n*} N_{t-1}^n (1 + \pi_m)(\tilde{\alpha} - \alpha^*) + \phi^m w^* l^{m*} N_{t-1}^n (J(\tilde{\alpha}) - J(\alpha^*)) \\
& + r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{98}$$

On the other hand, from the homogeneity of the production function and Euler's theorem, (98) becomes

$$\begin{aligned}
& F(R(\alpha^*) l^{n*} + \phi^m J(\alpha^*) l^{m*}) N_{t-1}^n, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^* N_{t-1}^n (1 + \alpha^*) \\
& + w^* l^{n*} N_{t-1}^n (1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
& + \phi^m w^* l^{m*} N_{t-1}^n l^{m*} (J(\tilde{\alpha}) - J(\alpha^*)) \\
& + r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{99}$$

Similarly, we have the following relationship. Thus, SP_t becomes as follows:

$$\begin{aligned}
SP_t = & F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& + (1 - \delta) a_{t-1} N_{t-1}^n (1 + \tilde{\alpha}) - \delta (s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m) \\
& - F(R(\alpha^*) l^{n*} N_{t-1}^n + \phi^m J(\alpha^*) l^{m*} N_{t-1}^n, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^* N_{t-1}^n (1 + \alpha^*)) \\
& + \delta (s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n + a^* N_{t-1}^n (1 + \alpha^*)) \\
& - w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& - \phi^m w^* l^{m*} N_{t-1}^n \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& + \tau_w^* w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& + \phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\tilde{\alpha}) - J(\alpha^*)) \\
& + \tau_r^* r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (1 + \pi_m) (g^{y,n} + g^{ind,n} + a^*) (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*) \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*) \\
& - a^* N_{t-1}^n (1 + \alpha^*)
\end{aligned} \tag{100}$$

Combining 4th line and 14th line, we have

$$\begin{aligned}
SP_t = & F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& + (1 - \delta) a_{t-1} N_{t-1}^n (1 + \tilde{\alpha}) - \delta (s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m) \\
& - F(R(\alpha^*) l^{n*} N_{t-1}^n + \phi^m J(\alpha^*) l^{m*} N_{t-1}^n, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^* N_{t-1}^n (1 + \alpha^*)) \\
& + \delta (s^{n*} N_{t-1}^n + s^{m*} \alpha^* N_{t-1}^n) - (1 - \delta) a^* N_{t-1}^n (1 + \alpha^*) \\
& - w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& - \phi^m w^* l^{m*} N_{t-1}^n \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& + \tau_w^* w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& + \phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\tilde{\alpha}) - J(\alpha^*)) \\
& + \tau_r^* r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (1 + \pi_m) (g^{y,n} + g^{ind,n} + a^*) (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*) \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*)
\end{aligned}$$

Note that $-(1-\delta)a^*N_{t-1}^n(1+\alpha^*) = -(1-\delta)a^*N_{t-1}^n(1+\tilde{\alpha}+\alpha^*-\tilde{\alpha}) = -(1-\delta)a^*N_{t-1}^n(1+\tilde{\alpha}) + (1-\delta)N_{t-1}^n(\tilde{\alpha}-\alpha^*)$. Also note that $\delta s^{n*}N_{t-1}^n$ is canceled out from the second and fourth lines. Also note that in the second line $N_{t-1}^m = N_{t-1}^n(\alpha^* + \tilde{\alpha} - \alpha^*)$. Thus,

$\delta s^{m*} N_{t-1}^n \alpha^*$ is canceled out from the second and fourth line. Thus, we have

$$\begin{aligned}
SP_t = & F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& + (1 - \delta) a_{t-1} N_{t-1}^n (1 + \tilde{\alpha}) \\
& - F(R(\alpha^*) l^{n*} N_{t-1}^n + \phi^m J(\alpha^*) l^{m*} N_{t-1}^n, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^*(N_{t-1}^n + N_{t-1}^n \alpha^*)) \\
& - (1 - \delta) a^* N_{t-1}^n (1 + \tilde{\alpha}) + (1 - \delta) a^* N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - \delta s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& - \phi^m w^* l^{m*} N_{t-1}^n \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& + \tau_w^* w^* l^{n*} N_{t-1}^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& + \phi^m \tau_w^* w^* l^{m*} N_{t-1}^n (J(\tilde{\alpha}) - J(\alpha^*)) \\
& + \tau_r^* r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (1 + \pi_m) (g^{y,n} + g^{ind,n} + a^*) (\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (g^{y,m} + g^{ind,m} + a^*) \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - N_{t-1}^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*)
\end{aligned}$$

We subtract and add $F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m))$ and $s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*)$ from and to SP_t . We also combine $s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*)$, $r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*)$

and $\tau_r^* s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*)$. Then, SP_t becomes as follows

$$\begin{aligned}
SP_t = & F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& - F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m)) \\
& + (1 - \delta) a_{t-1} N_{t-1}^n (1 + \tilde{\alpha}) - (1 - \delta) a^* N_{t-1}^n (1 + \tilde{\alpha}) \\
& + F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m)) \\
& - F(N_t^n R(\alpha^*) l^{n*} + \phi^m N_t^m J(\alpha^*) l^{m*}, s^{n*} N_t^n + s^{m*} N_t^m \alpha^* + a^*(N_{t-1}^n + \alpha^* N_{t-1}^m)) \\
& + (1 - \delta) a^* N_{t-1}^n (\tilde{\alpha} - \alpha^*) + (1 - \delta) s^{m*} N_{t-1}^n (\tilde{\alpha} - \alpha^*) \\
& - (1 - \tau_w^*) w^* l^{n*} N_t^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
& - (1 - \tau_w^*) \phi^m w^* l^{m*} N_t^m \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - (1 + (1 - \tau_r^*) r^*) s^{m*} N_t^n (\tilde{\alpha} - \alpha^*) \\
& - N_t^n (1 + \pi_m) (g^{y,n} + g^{ind,n} + a^*) (\tilde{\alpha} - \alpha^*) \\
& - N_t^n (g^{y,m} + g^{ind,m} + a^*) \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - N_t^n (b^{m*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{101}$$

Note that $N_t^n = N_{t-1}^n R(\tilde{\alpha})$ and $N_t^m = N_{t-1}^m J(\tilde{\alpha})$. Then, the first three lines of

(101) can be re-written as follows:

$$\begin{aligned}
& F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a_{t-1}(N_{t-1}^n + N_{t-1}^m)) \\
& - F(N_t^n l^{n*} + \phi^m N_t^m l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^m + a^*(N_{t-1}^n + N_{t-1}^m)) \\
& + (1 - \delta)a_{t-1}N_{t-1}^n(1 + \tilde{\alpha}) - (1 - \delta)a^*N_{t-1}^n(1 + \tilde{\alpha}) \\
& = F(N_{t-1}^n R(\tilde{\alpha})l^{n*} + \phi^m N_{t-1}^n J(\tilde{\alpha})l^{m*}, s^{n*} N_{t-1}^n + s^{m*} \tilde{\alpha} N_{t-1}^n + a_{t-1}(N_{t-1}^n + \tilde{\alpha} N_{t-1}^n)) \\
& - F(N_{t-1}^n R(\tilde{\alpha})l^{n*} + \phi^m N_{t-1}^n J(\tilde{\alpha})l^{m*}, s^{n*} N_{t-1}^n + s^{m*} \tilde{\alpha} N_{t-1}^n + a^*(N_{t-1}^n + \tilde{\alpha} N_{t-1}^n)) \\
& + (1 - \delta)a_{t-1}N_{t-1}^n(1 + \tilde{\alpha}) - (1 - \delta)a^*N_{t-1}^n(1 + \tilde{\alpha}) \\
& = N_{t-1}^n \{F(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, s^{n*} + s^{m*} \tilde{\alpha} + a_{t-1}(1 + \tilde{\alpha})) \\
& - F(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, s^{n*} + s^{m*} \tilde{\alpha} + a^*(1 + \tilde{\alpha})) \\
& + (1 - \delta)a_{t-1}(1 + \tilde{\alpha}) - (1 - \delta)a^*(1 + \tilde{\alpha})\} \\
& = N_{t-1}^n \int_{s^{n*} + s^{m*} \tilde{\alpha} + a^*(1 + \tilde{\alpha})}^{s^{n*} + s^{m*} \tilde{\alpha} + a_{t-1}(1 + \tilde{\alpha})} [F_K(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, z) + (1 - \delta)] dz \tag{102}
\end{aligned}$$

The fourth line and the fifth line of (101) can be transformed as follows

$$\begin{aligned}
& F(N_{t-1}^n R(\tilde{\alpha})l^{n*} + \phi^m N_{t-1}^n J(\tilde{\alpha})l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \tilde{\alpha} + a^*(N_{t-1}^n + N_{t-1}^n \tilde{\alpha})) \\
& - F(N_{t-1}^n R(\alpha^*)l^{n*} + \phi^m N_{t-1}^n J(\alpha^*)l^{m*}, s^{n*} N_{t-1}^n + s^{m*} N_{t-1}^n \alpha^* + a^*(N_{t-1}^n + \alpha^* N_{t-1}^n)) \\
& = N_{t-1}^n \{F(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, s^{n*} + s^{m*} \tilde{\alpha} + a^*(1 + \tilde{\alpha})) \\
& - F(R(\alpha^*)l^{n*} + \phi^m J(\alpha^*)l^{m*}, s^{n*} + s^{m*} \alpha^* + a^*(1 + \alpha^*)) \\
& = N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} [F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*} z + a^*(1 + z))(R'(z)l^{n*} + \phi^m J'(z)l^{m*}) \\
& + F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*} z + a^*(1 + z))(s^{m*} + a^*)] dz \tag{103}
\end{aligned}$$

. Then, SP_t becomes as follows:

$$\begin{aligned}
SP_t = & N_{t-1}^n \int_{s^{n*}+s^{m*}\tilde{\alpha}+a^*(1+\tilde{\alpha})}^{s^{n*}+s^{m*}\tilde{\alpha}+a_{t-1}(1+\tilde{\alpha})} [F_K(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, z) + (1 - \delta)] dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} [F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))(R'(z)l^{n*} + \phi^m J'(z)l^{m*}) \\
& + F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))(s^{m*} + a^*)] dz \\
& + (1 - \delta)N_{t-1}^n(a^* + s^{m*})(\tilde{\alpha} - \alpha^*) \\
& - (1 - \tau_w^*)w^*l^{n*}N_t^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
& - (1 - \tau_w^*)\phi^m w^*l^{m*}N_t^n\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - (1 + (1 - \tau_r^*)r^*)s^{m*}N_t^n(\tilde{\alpha} - \alpha^*) \\
& - N_t^n(1 + \pi_m)(g^{y,n} + g^{ind,n})(\tilde{\alpha} - \alpha^*) \\
& - N_t^n(g^{y,m} + g^{ind,m})\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - N_t^n(b^{m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{104}$$

Let $c^{o,m*}$ be the consumption of old immigrants at the initial steady state. From the individual budget constraint, $c^{o,m*} = b^m + (1 + (1 - \tau_r^*)r^*)s^{m*}$. Thus, SP_t becomes

$$\begin{aligned}
SP_t = & N_{t-1}^n \int_{s^{n*}+s^{m*}\tilde{\alpha}+a^*(1+\tilde{\alpha})}^{s^{n*}+s^{m*}\tilde{\alpha}+a_{t-1}(1+\tilde{\alpha})} [F_K(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, z) + (1-\delta)] dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} [\tilde{F}_L \times (R'(z)l^{n*} + \phi^m J'(z)l^{m*}) \\
& + \tilde{F}_K \times (s^{m*} + a^*)] dz \\
& + (1-\delta)N_{t-1}^n a^*(\tilde{\alpha} - \alpha^*) \\
& (1-\delta)s^{m*}N_{t-1}^n(\tilde{\alpha} - \alpha^*) \\
& - (1-\tau_w^*)w^*l^{n*}N_{t-1}^n(1+\pi_m)(\tilde{\alpha} - \alpha^*) \\
& - (1-\tau_w^*)\phi^m w^*l^{m*}N_{t-1}^n\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - (1+(1-\tau_r^*)r^*)s^{m*}N_{t-1}^n(\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n(1+\pi_m)(g^{y,n} + g^{ind,n} + a^*)(\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n(g^{y,m} + g^{ind,m} + a^*)\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
& - N_{t-1}^n(c^{o,m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{105}$$

where $\tilde{F}_L = F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))$

$\tilde{F}_K = F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1+z))$

From fourth line to twelfth line, we can rearrange as follows:

$$\begin{aligned}
& N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} \{(1-\delta)(s^{m*} + a^*) \\
& - R'(\alpha)(1-\tau_w^*)w^*l^{n*} - (1-\tau_w^*)\phi^m w^*l^{m*}J'(z) \\
& - R'(\alpha)(g^{y,n} + g^{ind,n} + a^*) \\
& - (g^{y,m} + g^{ind,m} + a^*)J'(z) \\
& - (c^{o,m*} + g^{o,m} + g^{ind,m})\} dz
\end{aligned} \tag{106}$$

Thus, SP_t becomes as follows:

$$\begin{aligned}
SP_t = & N_{t-1}^n \int_{s^{n*}+s^{m*}\tilde{\alpha}+a^*(1+\tilde{\alpha})}^{s^{n*}+s^{m*}\tilde{\alpha}+a_{t-1}(1+\tilde{\alpha})} [F_K(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, z) + (1 - \delta)] dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} \{ \tilde{F}_L \times (R'(z)l^{n*} + \phi^m J'(z)l^{m*}) \\
& + (\tilde{F}_K \times (s^{m*} + a^*) \\
& - R'(\alpha)(1 - \tau_w^*)w^*l^{n*} - (1 - \tau_w^*)\phi^m w^*l^{m*} J'(z) \\
& - R'(\alpha)(g^{y,n} + g^{ind,n} + a^*) - (g^{y,m} + g^{ind,m} + a^*)J'(z) - (c^{o,m*} + g^{o,m} + g^{ind,m}) \} dz
\end{aligned} \tag{107}$$

Note that $(1 - \tau_w^*)w^*l^{n*}$ is the after-tax income of the native when the native is young at the initial steady state. From the individual budget constraint, this is equal to $c^{y,n*} + s^{n*}$. Similarly, $(1 - \tau_w^*)\phi^m w^*l^{m*} = c^{y,m*} + s^{m*}$. Therefore, we have

$$\begin{aligned}
SP_t = & N_{t-1}^n \int_{s^{n*}+s^{m*}\tilde{\alpha}+a^*(1+\tilde{\alpha})}^{s^{n*}+s^{m*}\tilde{\alpha}+a_{t-1}(1+\tilde{\alpha})} [F_K(R(\tilde{\alpha})l^{n*} + \phi^m J(\tilde{\alpha})l^{m*}, z) + (1 - \delta)] dz \\
& + N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} \{ R'(z)[\tilde{F}_L l^{n*} - (c^{y,n*} + s^{n*} + g^{y,n} + g^{ind,n} + a^*)] \\
& + J'(z)[\tilde{F}_L \phi^m l^{m*} - (c^{y,m*} + s^{m*} + g^{y,m} + g^{ind,m} + a^*)] \\
& + (\tilde{F}_K + 1 - \delta)(s^{m*} + a^*) - (c^{o,m*} + g^{o,m} + g^{ind,m}) \} dz
\end{aligned} \tag{108}$$

where $\tilde{F}_L = F_L(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1 + z))$

$\tilde{F}_K = F_K(R(z)l^{n*} + \phi^m J(z)l^{m*}, s^{n*} + s^{m*}z + a^*(1 + z))$

In the above equation, the first line is the effect of increasing the government savings. The second line is the MPL condition for the native. The third line is the MPL condition for immigrants. The fourth line measure the intra-redistributional effect.

Appendix B5

Now, to see the correctness of the above equation, check what will happen to (??) when natives and immigrants have the same productivities and the same preferences. First, note that $l^n = l^m$ and $g^{y,n} = g^{y,m}$, $s^{n*} = s^{m*}$ and $\phi^m = 1$ when natives and immigrants have the same preferences and productivities. Thus, we have

$$\begin{aligned}
SP_t &= N_{t-1}^n \int_{(s^{n*}+a^*)(1+\tilde{\alpha})}^{(s^{n*}+a_{t-1})(1+\tilde{\alpha})} [F_K(R(\tilde{\alpha})l^{n*}(1+\tilde{\alpha}), z) + (1-\delta)] dz \\
&\quad + N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} R'(z) [\tilde{F}_L l^{n*} - (c^{y,n*} + s^{n*} + g^{y,n} + g^{ind,n} + a^*)] dz \\
&\quad + N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} J'(z) [\tilde{F}_L l^{n*} - (c^{y,n*} + s^{n*} + g^{y,n} + g^{ind,n} + a^*)] J'(z) dz \\
&\quad + N_{t-1}^n \times (s^{n*} + a^*) \int_{\alpha^*}^{\tilde{\alpha}} [\tilde{F}_K + 1 - \delta] dz \\
&\quad - N_{t-1}^n \int_{\alpha^*}^{\tilde{\alpha}} (c^{o,n*} + g^{o,n} + g^{ind,n}) dz
\end{aligned} \tag{109}$$

where $\tilde{F}_L = F_L(R(z)l^{n*} + J(z)l^{n*}, s^{n*} + s^{n*}z + a^*(1+z))$ and

$$\tilde{F}_K = F_K(R(z)l^{n*} + J(z)l^{n*}, s^{n*} + s^{n*}z + a^*(1+z))$$

The first line of (109) can be transformed as follows:

$$\begin{aligned}
&N_{t-1}^n \{ F(R(\tilde{\alpha})l^{n*}(1+\tilde{\alpha}), (s^{n*} + a_{t-1})(1+\tilde{\alpha})) + (1-\delta)(s^{n*} + a_{t-1})(1+\tilde{\alpha}) \\
&\quad - F(R(\tilde{\alpha})l^{n*}(1+\tilde{\alpha}), (s^{n*} + a^*)(1+\tilde{\alpha})) - (1-\delta)(s^{n*} + a^*)(1+\tilde{\alpha}) \} \\
&= N_{t-1}^n (1+\tilde{\alpha}) \{ \{ F(R(\tilde{\alpha})l^{n*}, (s^{n*} + a_{t-1})) + (1-\delta)(s^{n*} + a_{t-1}) \\
&\quad - F(R(\tilde{\alpha})l^{n*}, (s^{n*} + a^*)) - (1-\delta)(s^{n*} + a^*) \} \\
&= N_{t-1}^n (1+\tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1-\delta)] dz
\end{aligned}$$

Thus, 2nd to 4th line of (109) can be transformed as follows:

$$\begin{aligned}
& + N_{t-1}^n F(R(\tilde{\alpha})l^{n*} + J(\tilde{\alpha})l^{n*}, s^{n*} + s^{n*}\tilde{\alpha} + a^*(1 + \tilde{\alpha})) \\
& - N_{t-1}^n F(R(\alpha^*)l^{n*} + J(\alpha^*)l^{n*}, s^{n*} + s^{n*}\alpha^* + a^*(1 + \alpha^*)) \} \\
& + N_{t-1}^n (1 - \delta)(s^{n*} + a^*)(\tilde{\alpha} - \alpha^*) \\
& - (1 + \pi_m)N_{t-1}^n (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\tilde{\alpha} - \alpha^*) \\
& - N_{t-1}^n (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(J(\tilde{\alpha}) - J(\alpha^*)) \\
& - N_{t-1}^n (c^{o,n*} + g^{o,n} + g^{ind,n})(\tilde{\alpha} - \alpha^*)
\end{aligned}$$

On the other hand, from the resource constraint at the initial steady state we have

$$\begin{aligned}
& F(l^{n*} R(\alpha^*)(1 + \alpha^*)N_0, (s^* + a^*)(1 + \alpha^*)N_0 + (1 - \delta)(s^{n*} + a^*)N_0(1 + \alpha^*)N_0 \\
& = (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})N_0 R(\alpha^*)(1 + \alpha^*) \\
& + (c^{o,n*} + g^o + g^{ind})(1 + \alpha^*)
\end{aligned}$$

We divide the above resource constraint by $N_0(1 + \alpha^*)$ and multiply $N_{t-1}^n(\tilde{\alpha} - \alpha^*)$.

Then, we have

$$\begin{aligned}
& F(l^{n*} R(\alpha^*)N_{t-1}^n, (s^* + a^*)N_{t-1}(\tilde{\alpha} - \alpha^*) + (1 - \delta)(s^{n*} + a^*)N_{t-1}^n(\tilde{\alpha} - \alpha^*) \\
& = (1 + \alpha^*)R(\alpha^*)(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\tilde{\alpha} - \alpha^*) \\
& + (c^{o,n*} + g^o + g^{ind})N_{t-1}^n(\tilde{\alpha} - \alpha^*)
\end{aligned}$$

Solving for $-(c^{o,n*} + g^o + g^{ind})N_{t-1}^n(\tilde{\alpha} - \alpha^*)$, we have

$$\begin{aligned}
-(c^{o,n*} + g^o + g^{ind})N_{t-1}^n(\tilde{\alpha} - \alpha^*) &= -F(l^{n*}R(\alpha^*)N_{t-1}^n, (s^* + a^*)N_{t-1})(\tilde{\alpha} - \alpha^*) \\
&\quad - (1 - \delta)(s^{n*} + a^*)N_{t-1}^n(\tilde{\alpha} - \alpha^*) \\
&\quad + (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})N_{t-1}^nR(\alpha^*)(\tilde{\alpha} - \alpha^*) \\
&= -N_{t-1}^n\{F(l^{n*}R(\alpha^*), s^* + a^*)(\tilde{\alpha} - \alpha^*) \\
&\quad + (1 - \delta)(s^{n*} + a^*)(\tilde{\alpha} - \alpha^*) \\
&\quad - (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)(\tilde{\alpha} - \alpha^*)\}
\end{aligned}$$

Thus, SP_t becomes

$$\begin{aligned}
SP_t &= N_{t-1}^n(1 + \tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1 - \delta)]dz \\
&\quad + N_{t-1}^n\{F(R(\tilde{\alpha})l^{n*} + J(\tilde{\alpha})l^{n*}, s^{n*} + s^{n*}\tilde{\alpha} + a^*(1 + \tilde{\alpha})) \\
&\quad - F(R(\alpha^*)l^{n*} + J(\alpha^*)l^{n*}, s^{n*} + s^{n*}\alpha^* + a^*(1 + \alpha^*)) \\
&\quad - F(l^{n*}R(\alpha^*), s^* + a^*)(\tilde{\alpha} - \alpha^*) \\
&\quad - (1 + \pi_m)(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\tilde{\alpha} - \alpha^*) \\
&\quad - (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(J(\tilde{\alpha}) - J(\alpha^*)) \\
&\quad + (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)(\tilde{\alpha} - \alpha^*)\} \tag{110}
\end{aligned}$$

Note that $(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})J(\alpha^*)$ and $(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)\alpha^*$

are canceled out. Thus, SP_t becomes

$$\begin{aligned}
SP_t = & N_{t-1}^n (1 + \tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1 - \delta)] dz \\
& + N_{t-1}^n \{ F(R(\tilde{\alpha})l^{n*} + J(\tilde{\alpha})l^{n*}, s^{n*} + s^{n*}\tilde{\alpha} + a^*(1 + \tilde{\alpha})) \\
& - F(R(\alpha^*)l^{n*} + J(\alpha^*)l^{n*}, s^{n*} + s^{n*}\alpha^* + a^*(1 + \alpha^*)) \\
& - F(l^{n*}R(\alpha^*), s^* + a^*)(\tilde{\alpha} - \alpha^*) \\
& - (1 + \pi_m)(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\tilde{\alpha} - \alpha^*) \\
& - (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})J(\tilde{\alpha}) \\
& + (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)\tilde{\alpha} \} \tag{111}
\end{aligned}$$

Note that $-(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})J(\tilde{\alpha}) + (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})R(\alpha^*)\tilde{\alpha}$ become equal to

$$-(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(1 + \pi_m)\tilde{\alpha}(\tilde{\alpha} - \alpha^*)$$

Thus, SP_t becomes

$$\begin{aligned}
SP_t = & N_{t-1}^n (1 + \tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1 - \delta)] dz \\
& + N_{t-1}^n \{ F(R(\tilde{\alpha})l^{n*} + J(\tilde{\alpha})l^{n*}, s^{n*} + s^{n*}\tilde{\alpha} + a^*(1 + \tilde{\alpha})) \\
& - F(R(\alpha^*)l^{n*} + J(\alpha^*)l^{n*}, s^{n*} + s^{n*}\alpha^* + a^*(1 + \alpha^*)) \\
& - F(l^{n*}R(\alpha^*), s^* + a^*)(\tilde{\alpha} - \alpha^*) \\
& - (1 + \pi_m)(c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(\tilde{\alpha} - \alpha^*) \\
& - (c^{y,n*} + s^{n*} + a^* + g^y + g^{ind})(1 + \pi_m)\tilde{\alpha}(\tilde{\alpha} - \alpha^*) \} \tag{112}
\end{aligned}$$

The second line of (112) becomes

$$N_{t-1}^n (1 + \tilde{\alpha}) \{ F(R(\tilde{\alpha})l^{n*}, s^{n*} + a^*)$$

The third and fourth line of the above equations become

$$\begin{aligned}
& -N_{t-1}^n(1+\alpha^*)F(R(\alpha^*)l^{n*}, s^{n*}+a^*) \\
& -N_{t-1}^nF(l^{n*}R(\alpha^*), s^*+a^*)(\tilde{\alpha}-\alpha^*) \\
& = -N_{t-1}^n(1+\tilde{\alpha})F(R(\alpha^*)l^{n*}, s^{n*}+a^*)
\end{aligned}$$

The fifth and sixth line becomes

$$-(1+\pi_m)N_{t-1}^n(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(1+\tilde{\alpha})(\tilde{\alpha}-\alpha^*)$$

Thus, SP_t becomes

$$\begin{aligned}
SP_t &= N_{t-1}^n(1+\tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1-\delta)]dz \\
&+ N_{t-1}^{n*}(1+\tilde{\alpha})\{F(R(\tilde{\alpha})l^{n*}, s^{n*}+a^*) - F(R(\alpha^*)l^{n*}, s^{n*}+a^*) \\
&- (1+\pi_m)(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})(\tilde{\alpha}-\alpha^*)\} \\
&= N_{t-1}^n(1+\tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1-\delta)]dz \\
&+ N_{t-1}^n(1+\tilde{\alpha}) \int_{\alpha^*}^{\tilde{\alpha}} (1+\pi_m)l^{n*}F_L(R(z), s^{n*}+a^*)dz \\
&- N_{t-1}^n(1+\tilde{\alpha}) \int_{\alpha^*}^{\tilde{\alpha}} (1+\pi_m)(c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})dz
\end{aligned}$$

Combining the second and the third line of the above equation, we have

$$\begin{aligned}
SP_t &= N_{t-1}^n(1+\tilde{\alpha}) \int_{s^{n*}+a^*}^{s^{n*}+a_{t-1}} [F_K(R(\tilde{\alpha})l^{n*}, z) + (1-\delta)]dz \\
&+ N_{t-1}^n(1+\tilde{\alpha}) \int_{\alpha^*}^{\tilde{\alpha}} R'(z)[F_L(R(z), s^{n*}+a^*)l^{n*} \\
&- (c^{y,n*}+s^{n*}+a^*+g^y+g^{ind})]dz
\end{aligned} \tag{113}$$

This is SP_t when immigrant and native have the same productivities and preferences.