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Abstract

In this note, I investigate the preferences under which household public goods such as consumption of children will increase regardless of an initial equilibrium allocation when the wife's bargaining power increases and the household resource allocation is Pareto-efficient. I reformulate the responsiveness condition proposed by Blundell, Chiappori, and Meghir (Journal of Political Economy, 2005) and show that it is satisfied globally when the marginal utility of the husband's consumption is constant and that of the wife's consumption is diminishing. I also show other cases in which the consumption of children increases with an increase of the bargaining power of the wife. This suggests that in a large class of preferences, an increase in bargaining power of the wife will lead to an increase in household public goods regardless of initial equilibrium allocation.

1 Introduction

Consumption of children has the property of public goods in a household. When the welfare of children increases, it raises the utility of both the mother and father. When the mother is enjoying such an increase, it is difficult to exclude the father's enjoyment of the increased welfare of children.

In the empirical literature, an increase in bargaining power of the mother relative to that of the father is frequently observed to increase the consumption of children. In the economics literature, household resource allocation is often assumed to be Pareto-efficient from the repeated nature of resource allocation and the low cost of negotiation between family members. When the household resource allocation is assumed to be Pareto-efficient, one might want to know the conditions under which an increase in bargaining power of the mother increases the consumption of children. Since the welfare of children is considered to have the property of household public goods, this question might be interpreted as an attempt to identify the conditions under which household public goods would increase when the bargaining power of the mother increases relative to that of the father.[1] One might guess that the fact that a mother has a higher willingness to pay for her children's welfare is sufficient to show that an increase in bargaining power of the mother increases the consumption of children. However, Bergstrom (1995) and Blundell, Chiappori, and Meghir (2004) correctly point out that even if the mother has a higher willingness to pay for the children's welfare than the father, this does not imply that a stronger bargaining power of the mother leads to an increase of the consumption of children. Blundell, Chiappori, and Meghir (2005) proposed a responsiveness condition in the collective model under which an increase in higher weight on the mother's utility in a Pareto-efficient allocation leads to an increase in the consumption of children. Their responsiveness condition is intuitive and useful. However, sometimes researchers are interested in ascertaining the type of preferences under which an increase

[1]Bergstrom and Cornes (1981) analyze the effect of income distribution on the efficient level of public goods and show that when preferences have a Gorman form, the Pareto-efficient level of public goods is independent of income distribution.

in bargaining power of the mother will lead to an increase in consumption of children in a model other than the collective model. In addition, researchers are interested in identifying the conditions under which an increase in bargaining power of the mother will lead to an increase in consumption of children globally, that is, regardless of the initial equilibrium condition. Because of the empirical positive correlation between the bargaining power of the mother and consumption of children independent of the initial allocation, this is a natural question. In this note, I reformulate the responsiveness condition in terms of the utility function by using the utility possibility frontier (UPF). I show that when the marginal utility of consumption of the father is constant and that of the mother is diminishing, an increase in bargaining power of the wife will lead to an increase in the consumption of children.

2 Analysis

2.1 Main Results

Consider a household consisting of a husband, a wife, and children. I denote the husband, wife, and children as h, w, and k, respectively. Furthermore, let c_i be the consumption of member i, where $i = h, w, k$. The husband and wife have the utility function denoted as $U_j = u_j(c_j) + \alpha^{jk} u_k(c_k)$, where $u_k(c_k)$ is the utility of children from the consumption of c_k and α^{jk} is j's altruism on the utility of children; here, $j = h, w$. The household uses its income to satisfy the consumption of the husband, wife, and children and the budget constraint $\sum_{j=h,w} c_j + c_k = m$. Let τ be a parameter related to the bargaining power of the wife. I assume that a household solves the following maximization problem:

$$\text{main problem}$$

$$\max U_h^{1-\tau} U_w^{\tau} \tag{1}$$

$$\text{s.t.} \sum_{j=h,w} c_j + c_k = m \tag{2}$$

$$\text{where } U_h = u_h(c_h) + \alpha^{hk} u_k(c_k), \tag{3}$$

$$U_w = u_w(c_w) + \alpha^{wk} u_k(c_k). \tag{4}$$

My interest is in how an increase in τ affects the consumption of children, c_k[2]. Using a collective approach, Blundell, Chiappori, and Meghir (2004) proposed the following responsiveness condition:

$$\sum_{j=h,w} \frac{\partial \left[\frac{\alpha^{jk} u'_k(c_k)}{u'_j(c)} \right]}{\partial \rho_j} > 0 \tag{5}$$

where ρ_j is the share of member j from the expenditure on private goods. The above condition can be understood as follows. At the Pareto-efficient allocation, the amount of public goods is determined so that $\sum_{j=h,w} \frac{\alpha^{jk} u'_k(c_k)}{u'_j(c_j)} = 1$. Thus, if $\sum_{j=h,w} \frac{\alpha^{jk} u'_k(c_k)}{u'_j(c_j)}$ increases by changing the bargaining power τ, the public goods in the household will increase. Several interesting questions arise on the responsiveness condition. First, what are the preferences under which the above responsiveness condition is satisfied? Sometimes researchers are interested in a model other than the collective model. In that case, it is useful to examine the above condition without using the collective model. Second, what are the preferences under which the above condition is satisfied globally? For example, when τ decreases, the consumption of the husband increases but that of the wife decreases. This means that the willingness to pay for the public goods of the husband increases whereas that of the wife decreases. This implies that even if the wife's willingness to pay is initially higher than the husband's willingness, the latter might become higher than the former as τ decreases. Thus, it is important to identify the conditions under which the responsiveness condition holds globally.

In this paper, I reformulate the responsiveness condition. My results can be summarized by the following propositions.

[2]Note that although the objective function in the above problem is $U_h^{1-\tau} U_w^{\tau}$, I can replace it with $(U_h - V_h)(U_w - V_w)$, where V_h and V_w are the threat points of the husband and wife respectively and V_h and V_w are affected by τ. As can be seen below, this argument does not change even if I replace $U_h^{1-\tau} U_w^{\tau}$ with $(U_h - V_h)(U_w - V_w)$.

Proposition 1

If

$$\frac{\alpha^{hk}u_k'}{(u_h')^2}u_h'' - \frac{\alpha^{wk}u_k'}{(u_w')^2}u_w'' > 0, \tag{6}$$

an increase in τ will increase the consumption of children.

The proof is provided in the next subsection. Several forms of preferences satisfy (6) globally. One obvious case is that where the marginal utility of consumption of the husband is constant but that of the wife is diminishing. In such a case, $u_h'' = 0$ and $u_w'' < 0$. Thus, we have $\frac{\alpha^{hk}u_k'}{(u_h')^2}u_h'' - \frac{\alpha^{wk}u_k'}{(u_w')^2}u_w'' > 0$ and the following proposition:

Proposition 2

If the marginal utility of consumption of the husband is constant and that of the wife is diminishing, an increase in τ will increase the consumption of children globally.

Proposition 2 suggests that in a large class of preferences, the consumption of children will increase globally with an increase of the bargaining power of the wife. Another obvious case is when $\alpha^{hk} \leq 0$ and $\alpha^{wk} > 0$. In this case, (6) is satisfied.

Proposition 3

When the wife obtains the positive utility from the welfare of children but the husband obtains negative utility from the welfare of children, an increase in τ will increase the consumption of children globally.

The third case is the case where the utility function of the husband and the wife regarding their own consumption exhibit the same coefficient of the constant absolute risk aversion (CARA) and α^{wk}/α^{hk} is greater than a certain value. Let the coefficient of CARA be γ where $\gamma > 0$. Then, (6) becomes $\gamma u_k \left\{ \frac{\alpha^{wk}}{u_w} - \frac{\alpha^{hk}}{u_k} \right\} > 0$. Since $\gamma u_k > 0$, we have

$$\frac{\alpha^{wk}}{\alpha^{hk}} > \frac{u_w'}{u_h'} \tag{7}$$

Note that u_w' takes the maximum value when $c_w = 0$ and that u_h' takes the minimum value when $c_h = m$. Thus, if $\alpha^{wk}/\alpha^{hk} > u_w'(0)/u_h'(m)$, (7) is satisfied globally.

Proposition 4

When u_h and u_w have the same coefficient of the constant absolute risk aversion and $\alpha^{wk}/\alpha^{hk} > u_w'(0)/u_h'(m)$, an increase in τ will increase the consumption of children globally.

In the following subsection, I give the proof of the proposition 1.

2.2 Proof of Propositions

For the proof of proposition 1, I first characterize the utility possibility frontier(UPF) of (U_h , U_w). Next, given this UPF, I assume that the household chooses U_h and U_w to maximize (1).
(Step 1) To characterize the UPF of (U_h , U_w), consider the following problem that calculates the UPF:

UPF

$$\max u_h(c_h) + \alpha^{hk} u_k(c_k) \tag{8}$$

$$\text{s.t. } u_w(c_w) + \alpha^{wk} u_k(c_k) = \overline{U}_w \tag{9}$$

$$c_h + c_w + c_k = m \tag{10}$$

To solve the above UPF, first, from (10), I rewrite $c_w = m - c_h - c_k$. Then, I plug into (9) and solve for c_h. This implies that c_h becomes a function of c_k and $\overline{U}_w$; I denote this as $c_h(c_k, \overline{U}_w)$. I next plug $c_h(c_k, \overline{U}_w)$ into (8); now, I have the following unconstrained maximization problem:

$$U_h(\overline{U}_w) \equiv \max_{\{c_k\}} u_h(c_h(c_k; \overline{U}_w)) + \alpha^{hk} u_k(c_k). \tag{11}$$

The first-order condition becomes

$$u_h^{'} \frac{\partial c_h}{\partial c_k} + \alpha^{hk} u_k'(c_k) = 0. \tag{12}$$

Next, I need to calculate $\frac{\partial c_h}{\partial c_k}$. From the definition of (9), I have

$$u_w(m - c_h(c_k; \overline{U}_w) - c_k) + \alpha^{wk} u_k(c_k) \equiv \overline{U}_w. \tag{13}$$

Taking a derivative with respect to c_k on both sides, I have

$$\frac{\partial c_h}{\partial c_k} = \frac{\alpha^{wk} u_k'}{u_w'} - 1. \tag{14}$$

This implies that I have

$$\frac{\alpha^{hk}u_k'}{u_h'}+\frac{\alpha^{wk}u_k'}{u_w'}=1. \tag{15}$$

Equation (15) is the famous Samuelson condition for the amount of public goods. Now, I check the second-order condition (SOC) of (11). The second derivative of (11) becomes as follows:

$$\text{SOC}\equiv u_h^{''}\left[\frac{\partial c_h}{\partial c_k}\right]^2+u_h'\frac{\partial^2 c_h}{\partial c_k^2}+u_k^{''}(c_k). \tag{16}$$

To calculate the SOC, I need to first calculate $\frac{\partial^2 c_h}{\partial c_k^2}$. From (14), this is equal to

$$\begin{aligned}\frac{\partial^2 c_h}{\partial c_k^2}&=\frac{\alpha^{wk}u_k^{''}u_w'-\alpha^{wk}u_k'u_w''[-1-\frac{\partial c_h}{\partial c_k}]}{[u_w']^2} &(17)\\ &=\frac{\alpha^{wk}u_k^{''}u_w'+\alpha^{wk}u_k'u_w''[\frac{\alpha^{wk}u_k'}{u_w'}]}{[u_w']^2} &(18)\\ &<0.\end{aligned}$$

Therefore, SOC<0.

(Step 2) For the UPF, it is straightforward to show that it is concave to the origin. The household maximizes (1) given that (U_h,U_w) is on the UPF. Since the objective function (1) is convex to the origin, maximizing (1) subject to the UPF gives the unique solution.

(Step 3) Now, I conduct comparative statics on the parameter of the wife's bargaining power, τ. Note that when U_h is measured on the horizontal axis and U_w is measured on the vertical axis, the absolute value of the slope of the indifference curve of the objective function (1) is

$$\left|\frac{dU_w}{dU_h}\right|=\frac{U_h^{-\tau}}{U_w^{-(1-\tau)}}=\frac{U_w^{1-\tau}}{U_h^{\tau}}.$$

Thus, an increase in τ will decrease the absolute value of the slope of the indifference curve when evaluated at the same point of (U_h,U_w). This implies that (U_h,U_w) maximizes the objective function (1) given that the UPF will move to the north west while U_w will increase and U_h will decrease.

(Step 4) At step 4, I calculate how a move of (U_h,U_w) on the UPF affects the equilibrium c_k. By taking the total derivative of the first-order condition on c_k

(12) with respect to c_k and $\overline{U}_w$, I have

$$SOC \times dc_k + u_h'' \frac{\partial c_h}{\partial c_k} \frac{\partial c_h}{\partial \overline{U}_w} d\overline{U}_w + u_h' \frac{\partial}{\partial \overline{U}_w} \left[\frac{\partial c_h}{\partial c_k} \right] d\overline{U}_w = 0. \tag{19}$$

To evaluate (19), I need to first calculate $\frac{\partial}{\partial \overline{U}_w} \left[\frac{\partial c_h}{\partial c_k} \right]$ and $\frac{\partial c_h}{\partial \overline{U}_w}$. Note that from (15), $\frac{\partial c_h}{\partial c_k} = \frac{\alpha^{wk} u_k'}{u_w'} - 1$. Thus, I have

$$\frac{\partial}{\partial \overline{U}_w} \left[\frac{\partial c_h}{\partial c_k} \right] = \frac{\alpha^{wk} u_k'}{(u_w')^2} u_w'' \frac{\partial c_h}{\partial \overline{U}_w}. \tag{20}$$

With regard to $\frac{\partial c_h}{\partial \overline{U}_w}$, from (13), I have $u_w(m - c_h(c_k; \overline{U}_w) - c_k) + \alpha^{wk} u_k(c_k) \equiv \overline{U}_w$. From the implicit function theorem, I have

$$\frac{\partial c_h}{\partial \overline{U}_w} = -\frac{1}{u_w'}. \tag{21}$$

Thus, (19) becomes as follows:

$$SOC \times dc_k + u_h'' \frac{\partial c_h}{\partial c_k} \frac{\partial c_h}{\partial \overline{U}_w} d\overline{U}_w + u_h' \frac{\alpha^{wk} u_k'}{(u_w')^2} u_w'' \frac{\partial c_h}{\partial \overline{U}_w} d\overline{U}_w = 0. \tag{22}$$

Note that $\frac{\alpha^{hk} u_k'}{u_h'} + \frac{\alpha^{wk} u_k'}{u_w'} = 1$. Thus, $\frac{\partial c_h}{\partial c_k} = \frac{\alpha^{wk} u_k'}{u_w'} - 1 = -\frac{\alpha^{hk} u_k'}{u_h'}$. Therefore, I have

$$SOC \times dc_k + u_h''[-\frac{\alpha^{hk} u_k'}{u_h'}] \frac{\partial c_h}{\partial \overline{U}_w} d\overline{U}_w + u_h' \frac{\alpha^{wk} u_k'}{(u_w')^2} u_w'' \frac{\partial c_h}{\partial \overline{U}_w} d\overline{U}_w = 0. \tag{23}$$

This implies that

$$\frac{dc_k}{d\overline{U}_w} = \frac{u_h' \frac{\partial c_h}{\partial \overline{U}_w}}{SOC} \{ \frac{\alpha^{hk} u_k'}{(u_h')^2} u_h'' - \frac{\alpha^{wk} u_k'}{(u_w')^2} u_w'' \}. \tag{24}$$

Since SOC<0 and $\frac{\partial c_h}{\partial \overline{U}_w} < 0$, I have $\frac{dc_k}{d\overline{U}_w} > 0$ if and only if $\frac{\alpha^{hk} u_k'}{(u_h')^2} u_h'' - \frac{\alpha^{wk} u_k'}{(u_w')^2} u_w'' > 0$. Therefore, when $\frac{\alpha^{hk} u_k'}{(u_h')^2} u_h'' - \frac{\alpha^{wk} u_k'}{(u_w')^2} u_w''$, an increase in τ will increase the consumption of children.

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