

Tsukuba Economics Working Papers
No. 2018-002

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Idiosyncratic Returns from Investment and Incomplete Markets**

by

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February 2018

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Welfare-improving Consumption Tax in the Presence of Wage Tax under Idiosyncratic Returns from Investment and Incomplete Markets

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Forthcoming from

FinanzArchiv / Public Finance Analysis

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[†]The author appreciates the comments of Professor Toshihiro Ihori of the University of Tokyo, Professor Daiji Kawaguchi of Hitotsubashi University, the seminar participants of the Canadian Public Economics meeting, the International Institute of Public Finance and anonymous referee. The author also appreciates very helpful comments from the editor, Professor Schöb, which improved the exposition of this paper substantially and made this paper more transparent. This work was supported by a grant from the Japan Society for the Promotion of Science (JSPS KAKENHI Grant Number 15K03501) and the research fund of the University of Tsukuba. The author is responsible for any errors in this paper.

Abstract

In a standard multi-period model, consumption tax and wage tax are equivalent. I show that when a capital market is incomplete—in the sense that the rates of return from risky investments are idiosyncratic and there is no insurance for such idiosyncratic risk—the introduction of consumption tax in the presence of wage tax improves welfare. This holds true even in the presence of optimal or non-optimal capital income taxes. In the general equilibrium model, the optimal level of consumption tax is determined to balance the benefits of the risk-sharing effect and asset accumulation effect and the costs of postponing government revenue to later periods.

JEL keywords: incomplete capital market, risk sharing, consumption tax, optimal taxation

1 Introduction

In many developed countries, revenue from consumption tax tends to be substantial—in some cases, almost matching that derived from labor income tax. Given that tax revenue from consumption tax is important to the revenue streams of many countries, one might ask about the optimal ratio of consumption tax to labor income tax from the perspective of economic efficiency.

In a simple standard multi-period model, consumption tax and wage tax induce the same consumption and labor supply behavior and generate the same discounted value of tax revenue as long as the after-tax wage rate is the same.¹ Although the two taxes induce different personal savings behaviors—because of their different effects on the timing of taxation—they also lead to the same resource allocation if government savings are taken into consideration. This means that consumption tax becomes redundant in the presence of wage tax. Thus, a simple multi-period model cannot explain the optimal ratio of consumption and wage taxes.²

The optimal ratio between these two forms of taxes can be evaluated with respect to the liquidity constraints of the economy. Hubbard et al. (1986) analyze the impact of fiscal policy under liquidity constraints. When consumers face liquidity constraints due to imperfect financial markets, they prefer to pay tax in later periods rather than in earlier ones. This finding implies that consumption tax is more efficient than wage tax, since the tax liability of the latter occurs only when consumers are working, while that of the former is spread over the consumer's lifetime.

Tax evasion behavior also offers insight into the optimal tax ratio. Boadway et al. (1994) show that in the presence of tax evasion behavior that affects labor-derived income tax, the optimal combination of consumption tax and labor income tax is important.

The present study demonstrates that when a capital market is incomplete—in the sense that the rates of returns from risky investments are idiosyncratic among individ-

¹I define the after-tax wage rate as $w(1 - \tau_w)/(1 + \tau_c)$, where w is the wage rate, τ_w is the wage tax rate, and τ_c is the consumption tax rate.

²This conclusion does not change, even in the presence of a non-linear income tax system. The seminal studies by Atkinson and Stiglitz (1976, 1980) show that the introduction of price distortions between consumer goods is optimal if the preference between consumption goods and leisure is not weakly separable in the presence of non-linear income tax. However, consumption tax does not distort the relative price of consumption over multiple periods. Thus, although the Atkinson–Stiglitz theorem is appropriate for explaining the use of capital income tax, it does not address the optimal combination of consumption and wage taxes.

uals and that there is no insurance to insure idiosyncratic returns—the introduction of consumption tax, even in the presence of wage tax, can increase economic efficiency, although the government may initially use capital income tax to absorb idiosyncratic risks.

The importance of an incomplete capital market can be seen in the data and in the literature. First, the share of non-financial assets, whose stochastic rates of return are not likely to be insured, is not low. For example, Bucks et al. (2009) report that according to the 2004 US Survey of Consumer Finances, only 35.7 percent of all assets are held in the form of publicly traded financial assets, while the rest are held as non-financial assets such as self-owned businesses, self-occupied homes, and real estate. Second, Wolf (2002) reports that in the 1998 Survey of Consumer Finances, the share of residential homes, unincorporated businesses, and real estate within total assets is 56 percent. This fact implies that consumers buying those assets can face substantial risks, presumably with the trade-off of higher expected returns. In the macroeconomics literature, it has been shown that a model with entrepreneurs is more successful at replicating the wealth distribution in the United States as demonstrated by Cagetti and De Nardi (2006) and Quadrini (2000) among others. The macroeconomics literature also shows that the implication of idiosyncratic capital income risk is different from the implication of idiosyncratic labor income risk. Angeletos (2007) shows that higher idiosyncratic risk reduces the capital labor ratio, in sharp contrast to the result in the case of idiosyncratic labor income risk. Thus, given the importance of an incomplete capital market, exploring the implications of tax policy in the presence of such a market is crucial.

In the presence of an incomplete capital market, the use of consumption tax can increase risk sharing, because although the consumption tax rate is constant, consumers must pay a higher (lower) amount of tax when in a good (bad) state. Thus, the use of consumption tax can increase economic efficiency. However, two questions arise with respect to such an argument. The first question is whether the private insurance market is able to insure individual investment risks and whether social insurance through consumption tax is redundant. The second question is whether social insurance through consumption tax is redundant when the government can instead use capital income tax, because the source of risk is the returns from risky investments, and capital income tax can absorb such risks. For the first question, I assume that the capital market for investors is incomplete in the sense that no insurance insures idiosyncratic returns from

risky investments. Insuring this type of investment risk would pose both a moral hazard problem and an adverse selection problem. For example, if there is no pressure to find a profitable investment opportunity because the insurance market insures investment risks, people would not work hard to find profitable investment opportunities.

The present study investigates the issue inherent in the second question. In the analysis presented here, I demonstrate that even if the government were to use capital income tax to absorb idiosyncratic risks, the use of consumption tax would increase economic efficiency. At first glance, readers might wonder why the use of consumption tax would increase economic efficiency in the presence of the idiosyncratic risks of risky investments and capital income tax. Since capital income tax can be used to absorb those idiosyncratic risks, readers might think that the use of consumption tax is redundant as a risk-sharing mechanism.

However, I argue that the use of consumption tax increases economic efficiency, even when the government initially uses wage tax and capital income tax; that is because consumption tax and wage tax each impose a tax burden, but at different times. The basic intuition can be understood as follows. When a capital market is incomplete, the expected rates of return from idiosyncratic risky investments are higher than those from safe investments. This is because from an individual point of view, the returns from idiosyncratic risky investments are stochastic, whereas from the macroeconomics viewpoint, risk from idiosyncratic risky investments does not exist. Thus, from a social planning perspective, it is desirable to increase investment in risky assets while simultaneously sharing those risks because the expected rates of return from risky investments are higher than those from safe investments. When the government starts to increase the consumption tax rate and reduce the wage tax rate by small amounts, it will shift the tax burden from working periods to all periods, because wage tax will impose a tax burden only when an individual works, while consumption tax will impose a tax burden when an individual consumes. Corresponding to this change in the timing of the tax burden, an individual will increase his or her savings, including investment in risky assets. Since the expected rates of return from risky investments are higher than those from safe assets and those risks are idiosyncratic, there is an efficiency gain from increasing the consumption tax rate and reducing the wage tax rate. Clearly, this effect of the shift from wage tax to consumption tax does not disappear, even in the presence of capital income tax.³

³The logic of this argument has similar flavor to the logic of the famous Domar-Musgrave effect

A natural question to the above argument is how the optimal level of consumption tax is determined in the presence of wage tax. As the government increases the consumption tax rate and simultaneously decreases the wage tax rate holding the after-tax wage rate constant, investment in safe assets and risky assets increases. In the partial equilibrium model where the factor accumulation does not affect the factor prices, maximizing the consumption tax rate is optimal. This implies that if there is no institutional upper limit on the consumption tax rate, there is no optimum. However, in a general equilibrium model, the situation is different. As the government increases the consumption tax rate and decreases the wage tax rate while holding the after-tax wage rate constant, individuals increase investment in safe assets and risky assets. On the other hand, in terms of government revenue, some tax revenue is postponed to later periods because of the different timing of the tax burden of wage tax and consumption tax. This implies that the government needs to issue government bonds. Furthermore, an increase in investment in safe assets by individuals is insufficient to absorb the increased issue of government bond. This implies that a shift from wage tax to consumption tax will affect the asset portfolio of the economy and affect the factor prices. In Section 2.3, I show that at the optimum, consumption tax is determined to balance the benefit of risk sharing and the cost due to changes in the factor prices.

This study is related to two strands of the economics literature. First, it relates to the literature that discuss what types of taxes work as social insurance. For example, Eaton and Rosen (1980) show that wage tax acts as insurance when the return from human capital is uncertain. Kopczuk (2003) demonstrates that inheritance tax provides social security for the rich. Conesa and Krueger (2006) calculate the optimal tax progression, focusing on the social insurance role of taxation.⁴ Jacobs et al. (2012) analyze the role of labor income tax and education subsidy when human capital ac-

(Domar and Musgrave, 1944). In the Domar-Musgrave model, the imposition of a proportional income tax with full loss offset increases private risk taking since the government shares the risk through the proportional income tax. In my model, the government encourages individuals to take more risk through a change of the timing of the imposition of taxes. I appreciate the editor, Ronnie Schöb, for pointing out the similarity of my result to the famous Domar-Musgrave effect.

⁴In the public finance literature, there are several other studies. Diamond (2005) analyzes various social insurance and tax policies with respect to the life-cycle when the market is incomplete. Boadway and Sato (2011) analyze optimal taxation in the presence of risky earnings, using the framework of Mirrlees's model (Mirrlees, 1971). Nishiyama and Smetters (2005) examine the effect of replacing income tax with consumption tax under idiosyncratic earnings shocks and an uncertain lifespan. Gottardi et al. (2016) analyze the optimal taxation of labor income and capital income in the general equilibrium model with production.

cumulation involves idiosyncratic risks. However, they do not analyze the issue of the optimal combination of wage tax and consumption tax. To the best of my knowledge, no study has thus far analyzed the optimal combination of consumption tax and wage tax in the presence of idiosyncratic risks. This study is the first to show that in the presence of idiosyncratic risk, consumption tax plays an important role even in the presence of wage tax and capital income tax.⁵

This study also relates to the growing body of the macroeconomics literature on incomplete capital markets. Angeletos (2007) analyzes capital accumulation in an economy where the return from capital is idiosyncratic and there is no insurance for such idiosyncratic returns. Kitao (2008) examines the effect of capital income tax in the presence of idiosyncratic capital income risk. However, none of these studies analyzes the effect of consumption tax in the presence of wage tax and capital income tax. I show that the use of consumption tax increases economic efficiency even in the presence of wage tax and capital income tax when the market is incomplete. I also characterize the optimal level of consumption tax in a general equilibrium model.

The remainder of this paper is organized as follows. In Section 2.1, I show that the introduction of consumption tax in the presence of wage tax improves welfare. In Section 2.2, I extend the analysis to the general equilibrium model and characterize the optimal level of consumption tax. I show that at the optimum, the benefits of the risk-sharing effect and increasing asset accumulation effect of consumption tax and the costs of postponing government revenue need to be balanced. In Section 3, the conclusions are presented and issues to be addressed in future research are discussed.

2 Analysis

2.1 The Effect of Consumption Tax in the Presence of Wage Tax

Consider an economy in which a continuum of agents with identical preferences live for two periods. The agents work only in the first period and save for consumption in the second period. The investment opportunities comprise one safe asset and several risky assets. Each agent looking for profitable investments allocates his or her savings between the one safe asset and one of the risky assets. In this economy, technology

⁵The model of Jacobs et al. (2012) assumes that in the first period, individuals do not work and they work in the second period. Thus, they focus primarily on the earlier stage of individuals, while in my model I assume that individuals work in the first period and they are retired in the second period. Thus, my model primarily focuses on life-cycle savings and consumption behavior.

is linear; the rate of return from the safe asset is r and the stochastic rate of return from those risky assets is \tilde{r} . In this study, I use $\tilde{\cdot}$ to denote that a variable with $\tilde{\cdot}$ is a random variable. It is assumed that $\tilde{r} = \tilde{a} \times F_I \times e$, where \tilde{a} is a random variable and $e \in [1, 2]$ is the individual's effort level (e.g., looking for profitable investments), which affects the return on investment of the chosen risky asset. I assume that e is chosen before the realization of the stochastic shock. F_I , which is assumed to be constant in this subsection, is the marginal product of the risky asset.⁶ I assume that the random variable \tilde{a} is independently and identically distributed (i.i.d.) across individuals and that the level of individual effort to look for a profitable investment opportunity is not observable by the public. This lack of observability prevents private financial institutions from sharing the investment risk. The information on profitable investment is not available to the government. Thus, the government cannot invest in risky assets. I assume that the expected rate of return from risky investments is always greater than that from safe assets as long as the agent makes a positive effort to look for profitable investment opportunities.

The preference of an agent is represented by the following additive and separable utility function:

$$u(c_1) - h(l) - v(e) + \frac{1}{1 + \rho} E[u(\tilde{c}_2)] \quad (1)$$

$$\text{where } u' > 0, u'' < 0, h' > 0, h'' > 0, v' > 0, v'' > 0$$

where (l, e, c_1, c_2) represent labor supply, effort level, consumption in period 1, and consumption in period 2, respectively. From the linear technology assumption, the return on investment and the wage rate are independent of labor supply and the amount of investment.

I assume that the government institutes linear consumption tax, linear wage tax, linear capital income taxes on the return from risky investments and the return from safe asset. Let s and i be the amount of investment in the safe asset and the risky asset, respectively. Let τ_w and τ_c be the rate of wage tax and the rate of consumption tax. Let τ_{ks} and τ_{kr} be the capital income tax rate on the return from the safe investment

⁶In Section 2.2, I consider the case where F_I is endogenous.

and risky investment, respectively. Then, the budget constraint is as follows:

$$(1 + \tau_c)c = wl(1 - \tau_w) - s - i \quad (2)$$

$$(1 + \tau_c)c_2 = s(1 + r(1 - \tau_{ks})) + i(1 + \tilde{r}(1 - \tau_{kr})) \quad (3)$$

Let Ω be the after-tax wage rate where $\Omega = w(1 - \tau_w)/(1 + \tau_c)$. Given the capital income tax rate, wage tax rate, and consumption tax rate, the individual optimization problem is as follows:

$$\max_{s,i,l,e} u\left(\Omega l - \frac{s}{1 + \tau_c} - \frac{i}{1 + \tau_c}\right) - h(l) - v(e) + \frac{1}{1 + \rho} E\left[u\left(\frac{s(1 + r(1 - \tau_{ks})) + i(1 + \tilde{r}(1 - \tau_{kr}))}{(1 + \tau_c)}\right)\right]$$

where $\Omega = w(1 - \tau_w)/(1 + \tau_c)$ (4)

The first-order conditions of l , s , i , and e are

$$\Omega u'(c_1) = h'(l), \quad (5)$$

$$u'(c_1) = \frac{1}{1 + \rho} E[u'(\tilde{c}_2)(1 + r(1 - \tau_{ks}))] \quad (6)$$

$$u'(c_1) = \frac{1}{1 + \rho} E[u'(\tilde{c}_2)(1 + \tilde{r}(1 - \tau_{kr}))] \quad (7)$$

$$\text{and } v'(e) = \frac{1}{1 + \rho} E\left[u'(\tilde{c}_2) \frac{d\tilde{r}}{de} \frac{i}{1 + t_c} (1 - \tau_{kr})\right] \quad (8)$$

$$\text{where } c_1 = \Omega l - \frac{i}{1 + \tau_c} - \frac{s}{1 + \tau_c} \text{ and } \tilde{c}_2 = \frac{s(1 + r(1 - \tau_{ks})) + i(1 + \tilde{r}(1 - \tau_{kr}))}{(1 + t_c)}$$

These first-order conditions show three important facts. First, as long as the after-tax wage rate, Ω , stays constant, an increase in τ_c does not affect labor supply but it increases investment in the safe asset and the risky asset by a factor of $1 + \tau_c$. Second, as long as the after-tax wage rate stays constant, an increase in τ_c does not change indirect utility. This is because $i/(1 + \tau_c)$ and $s/(1 + \tau_c)$ stay constant, which implies that the value of c_1 and c_2 do not change.

Now, consider a situation where there is no consumption tax but only a wage tax initially with some capital income taxes. Then, suppose that the government increases the consumption tax rate by τ_c from $\tau_c = 0$ and decreases the wage tax rate by $\tau_c(1 - \tau_w)$. In this case, the after-tax wage rate stays constant at $w(1 - \tau_w)$.⁷ With

⁷The after-tax wage rate becomes $w(1 - \tau_w + \tau_c(1 - \tau_w))/(1 + \tau_c) = w(1 - \tau_w)$. Thus, the after-tax wage rate is the same as before.

wage tax and some capital income taxes, the present value of government revenue is equal to $\tau_w wl + \{\tau_{ks}rs + \tau_{kr}\mu i\} (1+r)^{-1}$.⁸ Denote $E[\tilde{r}|e]$ as μ . When the wage tax rate is $\tau_w - \tau_c(1 - \tau_w)$ and the consumption tax rate is τ_c , the present value of the government revenue is as follows:

$$\begin{aligned} & wl(\tau_w - \tau_c(1 - \tau_w)) + \tau_c \left\{ c_1 + \frac{E[c_2]}{1+r} \right\} + \frac{1}{1+r} \{ \tau_{ks}rs + \tau_{kr}\mu i \} \\ & = wl\tau_w + \frac{\tau_c}{1+\tau_c}(\mu - r)i\frac{1}{1+r} + \tau_{ks}\frac{rs}{1+\tau_c} + \tau_{kr}\frac{\mu i}{1+\tau_c} \end{aligned} \quad (9)$$

Note that τ_w is the wage tax rate when the consumption tax rate is equal to zero. $\tau_w - \tau_c(1 - \tau_w)$ is the wage tax rate when the consumption tax rate increases from zero to τ_c . From (5)-(8), labor supply and effort level do not change as long as the after-tax wage rate stays constant when τ_c increases. Thus, the first term of (9) is not affected by τ_c .

The third term and the fourth term of (9) stay constant since $i/(1+\tau_c)$ and $s/(1+\tau_c)$ stay constant.

The second term of (9) is an increasing function of τ_c because $i/(1 + \tau_c)$ stays constant. Thus, government revenue increases. On the other hand, consumer utility does not change from the optimization problem of (4) as long as the after-tax wage rate stays constant. Thus, the government can increase the tax revenue while holding the utility of the consumer constant. Hence, there is a welfare gain by increasing the consumption tax rate and decreasing the wage tax rate with the after-tax wage rate held constant.⁹

The reader might wonder where the source of the welfare gain from increasing the consumption tax rate lies. As I demonstrated, consumption and labor supply remain the same when the government increases the consumption tax holding the after-tax wage rate constant. Thus, the reader might think that resource allocation in this economy is not affected when the government increases the consumption tax rate holding the after-tax wage rate constant.

However, when the consumption tax rate is increased, resource allocation is not the same. Note that in this economy, there is no macroeconomic risk and the return

⁸There is no macroeconomic risk from the viewpoint of the government. Thus, the government can discount the revenue from all sources in the second period by the risk-free interest rate.

⁹Note that we do not need to divide government revenue by $1+\tau_c$ when evaluate it. The government is the entity that imposes the consumption tax. When the government buys consumption goods from the market by using its government revenue, the government pays the consumption tax to itself.

from the risky investment is idiosyncratic. As a result, the equilibrium level of the risky investment is smaller than its social optimal level because the idiosyncratic return from the risky investment is not insured. An increase of consumption tax rate and a decrease of wage tax rate will increase investment in the risky asset because these changes of taxes will shift the tax burden from the first period to the second period and because the agents attempt to pay the increased tax burden in the second period by increasing investment in the safe asset and the risky asset. This increase in the risky investment is the fundamental source of the welfare gain. On the surface, the effect of this change in resource allocation shows up as an increase in tax revenue.¹⁰

The second term of (9), $\tau_c(1 + \tau_c)^{-1}i(\mu - r)i(1 + r)^{-1}$, has a special economic meaning. Let i be the amount of risky investment when the consumption tax rate is equal to τ_c . The amount of risky investment is $i/(1 + \tau_c)$ when the consumption tax rate is equal to zero. The increase in the risky investment is $i - i/(1 + \tau_c) = \tau_c(1 + \tau_c)^{-1}i$. Borrowing one dollar and investing it in the risky investment will result in a social benefit equal to $\mu - r$. Thus, the present social value of the increased investment in the risky asset due to an increase in the consumption tax rate is equal to $\tau_c(1 + \tau_c)^{-1}i(\mu - r)(1 + r)^{-1}$.

Proposition 1: Assume that the return from investment is idiosyncratic and the capital market is incomplete in the sense that the expected rate of return is greater than the risk-free interest rate. Assume that initially the government uses the wage tax and capital income tax but no consumption tax. Now assume that starting from $\tau_c = 0$, the government increases the consumption tax rate by τ_c and decreases the wage tax rate to keep the real wage rate constant. Then, tax revenue will increase while the consumer utility stays constant, thus bringing about a welfare gain equal to $\frac{\tau_c}{1 + \tau_c} \frac{(\mu - r)}{1 + r} i$.

Proposition 1 needs some clarification. First, when the capital market is complete and $\mu = r$, a shift in the consumption tax from the wage tax does not increase utility. Thus, an incomplete capital market is a crucial requirement for my result. Second, readers might argue that the above argument is based on a partial equilibrium analysis, which assumes that factor prices are fixed. In a model where prices are fixed, as in the

¹⁰In my thought experiment, I changed the tax rates so that the utility of the agent stays constant and examined whether a change in the consumption tax can increase government revenue. Of course, I can conduct a different thought experiment where the government changes the consumption tax rate with a fixed government revenue constraint. I can show that the utility of the consumer will increase when the government raises the consumption tax rate holding its revenue constraint. In either way, the intuition is the same.

current model, setting the consumption tax rate as high as possible and lowering the wage tax while holding the after-tax wage rate, Ω , constant will maximize the welfare of consumers. Thus, if the consumption tax rate has no upper limit, no optimum can be achieved. However, the general equilibrium model, where factor prices are affected by the factor endowment, has an optimal level of consumption tax. As the government increases the consumption tax rate, while decreasing the wage tax rate so that the after-tax wage rate stays constant, consumers increase investment in both safe assets and risky assets. When the government introduces consumption tax and decreases the wage tax rate, some government revenue comes in the second period. This implies that the government needs to issue government bonds. Since consumers increase investment in both safe and risky assets when the government issues government bonds, the relatively risky asset increases and the net safe asset decreases in the capital market. This will change factor prices and government revenue. Thus, at the optimum, the optimal consumption tax rate is determined to balance the welfare gain from risk sharing and the decrease in government revenue due to a change in the factor prices. In Section 2.2, I will show this result by using the general equilibrium model where the factor prices are determined endogenously.

2.2 General Equilibrium Model and the Optimum level of the Consumption Tax Rate

In this subsection, I analyze the effect of introducing consumption tax in the general equilibrium model and characterize the optimal level of the consumption tax rate.

Consider the overlapping generation model. In this economy, individuals live for two periods. In the first period, individuals work. At the end of the first period, they retire and in the second period, they do not work. In the first period, they can save in both safe assets and risky assets. In the second period, individuals receive the return from the safe and risky investments. The population growth rate is equal to n . I assume that the utility function is the same as in the previous subsection. Regarding government policy, I assume that the government uses wage tax, consumption tax, and capital income tax. I also assume that it can issue government bonds. Let D_t be the government debt issued in period t divided by the number of cohorts t . I assume that in the next period, the government needs to pay $(1 + r_{t+1})D_tN_t$.

The rate of return from the safe investment in period t is r_t and the rate of return

from the risky investment is \tilde{r}_t . The wage rate, the rate of return from the safe asset, and the rate of return from the risky asset depend on the total amount of labor, safe assets, risky assets, and effort level. I assume that there is no depreciation on both types of assets. Let $Y(L_t, S_{t-1}, I_{t-1})$ be the aggregate production function, where $L_t = N_t l_t$, $S_{t-1} = N_{t-1} s_{t-1} - N_{t-1} D_{t-1}$ and $I_{t-1} = N_{t-1} i_{t-1} e_{t-1} E[\tilde{a}]$. I assume that $Y(L_t, S_{t-1}, I_{t-1})$ exhibits constant returns to scale with respect to L_t, S_{t-1} and I_{t-1} and that $E[\tilde{a}] = 1$. I assume that the factor prices are determined as follows:

$$w_t = F_L(L_t, S_{t-1}, I_{t-1}), \quad r_t = F_S(L_t, S_{t-1}, I_{t-1}), \quad (10)$$

$$\tilde{r}_t = F_I(L_t, S_{t-1}, I_{t-1}) \times e_{t-1} \times \tilde{a} \quad \text{where } E[\tilde{a}] = 1 \quad E[\tilde{r}_t] = \mu_t \quad (11)$$

Denote individuals born in period t as cohort t and the population of cohort t as N_t . Denote $w_t(1 - \tau_{wt})$ as \hat{w}_t , where τ_{wt} is the wage tax rate imposed on cohort t . Let τ_{ct} , τ_{kst} , and τ_{krt} be the consumption tax rate of cohort t , capital income tax rate imposed on the return from the safe asset of cohort t , and capital income tax rate imposed on the return from the risky asset of cohort t . I assume that tax policies depend on the index of cohorts. My results hold even when I assume that policies are time-invariant and when I focus on stationary policies.

Define the indirect utility function as follows:

$$\begin{aligned} V(\hat{w}_t, \tau_{ct}, \tau_{ks}, \tau_{kr}) \equiv & \max_{l_t, s_t, i_t, e_t} u\left(\frac{\hat{w}_t l_t - i_t - s_t}{1 + \tau_{ct}}\right) - h(l_t) - v(e_t) \\ & + \frac{1}{1 + \rho} E\left[u\left(\frac{(1 + r_{t+1}(1 - \tau_{kst}))s_t + (1 + \tilde{r}_{t+1}(1 - \tau_{krt}))i_t}{1 + \tau_{ct}}\right)\right] \end{aligned} \quad (12)$$

Note that from the first-order conditions, we have

$$\begin{aligned} \frac{\partial V}{\partial \tau_{ct}} &= -\frac{\partial V}{\partial \hat{w}_t} \Omega; \quad \frac{\partial l_t}{\partial \tau_{ct}} = -\frac{\partial l_t}{\partial \hat{w}_t} \Omega; \quad \frac{\partial i_t}{\partial \tau_{ct}} = -\frac{\partial i_t}{\partial \hat{w}_t} \Omega + i; \\ \frac{\partial s_t}{\partial \tau_{ct}} &= -\frac{\partial s_t}{\partial \hat{w}_t} \Omega + i; \quad \text{where } \Omega = \hat{w}_t / (1 + \tau_{ct}) \end{aligned}$$

Tax revenue in period t is

$$\begin{aligned} R_t \equiv & (w - \hat{w}_t) l_t N_t + \tau_{ct} \frac{(\hat{w}_t l_t - i_t - s_t)}{1 + \tau_{ct}} N_t + \tau_{ct-1} \frac{(1 + r_t(1 - \tau_{kst-1}))s_{t-1} + (1 + \mu_t(1 - \tau_{krt-1}))i_{t-1}}{1 + \tau_{ct-1}} N_{t-1} \\ & + \tau_{kst-1} r_t s_{t-1} N_{t-1} + \tau_{krt-1} \mu_t i_{t-1} N_{t-1} \end{aligned}$$

The government budget constraint in period t is

$$R_t - (1 + r_t)D_{t-1}N_{t-1} + D_tN_t - (N_t + N_{t-1})G \geq 0 \quad (13)$$

where G is per capita government expenditure, which is assumed to be exogenous and fixed.

Let the intergenerational social discount factor be θ , where $\theta < 1$. Then, the problem of the government is to solve the following constrained optimization problem:

$$\begin{aligned} \max_{\hat{w}_t, \tau_{ct}, \tau_{kst}, \tau_{ksr}} \quad & \sum_{t=0}^{\infty} \theta^t N_t V(\hat{w}_t, \tau_{ct}, \tau_{ks}, \tau_{kr}) \\ \text{s.t.} \quad & R_t - (1 + r_t)D_{t-1}N_{t-1} + D_tN_t - (N_t + N_{t-1})G \geq 0 \end{aligned}$$

Note that when the government solves the above programming problem, it takes into consideration the fact that the factor prices are endogenous and determined by equations (10) and (11). Let Q be a Lagrangian function and β_t be the Lagrangian multiplier of the government budget constraint. Then, the Lagrangian function becomes as follows:

$$\begin{aligned} Q = \quad & \sum_{t=0}^{\infty} \theta^t N_t V(\hat{w}_t, \tau_{ct}, \tau_{ks}, \tau_{kr}) \\ & + \sum_{t=0}^{\infty} \beta_t (R_t - (1 + r_t)D_{t-1}N_{t-1} + D_tN_t) \end{aligned}$$

The first-order condition of D_tN_t becomes as follows:

$$D_tN_t : \beta_t = \beta_{t+1} \left(1 + r_{t+1} + \frac{\partial R_{t+1}}{\partial (D_tN_t)} \right)$$

where $\partial R_{t+1}/\partial (D_tN_t)$ shows the extent to which tax revenue changes through a change in the factor prices due to a change in the safe asset level. In the above first-order conditions, β_t is the marginal utility of having additional government revenue. $1 + r_{t+1}$ is the interest payment and principal of issuing one unit of government bonds. $\partial R_{t+1}/\partial (D_tN_t)$ measures the revenue cost of issuing government bonds in terms of the decreased government revenue in the next period. When the government issues government bonds, the accumulation of safe assets decreases. This will affect the

factor prices in the next period and also reduces tax revenue in the next period. Thus, $1 + r_{t+1} + \partial R_{t+1}/\partial(D_t N_t)$ measures the total cost of issuing government bonds. The first-order condition of $D_t N_t$ shows that the ratio of the marginal utility of additional government revenue in period t to that in period $t+1$ is equal to the total cost of issuing government bonds.

Now, the first-order condition of τ_{ct} is

$$\begin{aligned} & \theta^t N_t \frac{\partial V}{\partial \tau_{ct}} + \beta_t \frac{1}{(1 + \tau_{ct})^2} (\widehat{w}_t l_t - i_t - s_t) N_t \\ & + \beta_{t+1} \frac{1}{(1 + \tau_{ct})^2} (1 + r_{t+1} (1 - \tau_{kst})) s_t + (1 + \mu_{t+1} (1 - \tau_{krt})) i_t N_t \\ & + \frac{\partial l_t}{\partial \tau_{ct}} \frac{\partial Q}{\partial l_t} + \frac{\partial i_t}{\partial \tau_{ct}} \frac{\partial Q}{\partial i_t} + \frac{\partial s_t}{\partial \tau_{ct}} \frac{\partial Q}{\partial s_t} = 0. \end{aligned} \quad (14)$$

By using the first-order conditions of \widehat{w}_t and $D_t N_t$, the first-order condition of τ_{ct} can be written as follows:

$$\begin{aligned} & \underbrace{\beta_t \frac{1}{(1 + \tau_{ct})^2} \frac{\mu_{t+1} - r_{t+1}}{1 + r_{t+1}} N_t}_{\text{risk sharing effect}} + \underbrace{s_t \frac{\partial Q}{\partial s_t} + i_t \frac{\partial Q}{\partial i_t}}_{\text{effect through the increase of } s \text{ and } i} \\ & + \underbrace{\beta_{t+1} \frac{\partial R_{t+1}}{\partial(D_t N_t)} \frac{1}{1 + r_{t+1}} \frac{(1 + r_{t+1} (1 - \tau_{kst})) s_t + (1 + \mu_{t+1} (1 - \tau_{krt})) i_t}{(1 + \tau_{ct})^2} N_t}_{\text{the effect through increased government debt}} = 0 \end{aligned} \quad (15)$$

The first term of (15) is the one we saw in equation (9) in the previous subsection. We assume that the rate of return from the risky investment is higher than r because of the incompleteness of the capital market. On the contrary, from a macroeconomics point of view, there is no aggregate risk. Thus, when consumption tax increases, individuals will increase investment in risky assets and safe assets to pay the tax burden in the second period. Since the rate of return from risky investments is higher than the risk-free interest rate but there is no aggregate risk, there is a welfare gain by increasing the consumption tax rate. The second and third terms are the effects on tax revenue in the next period. When consumption tax increases, individuals increase investment in risky assets and safe assets. A change in the production factors then changes the factor prices and the tax revenue of the government. The fourth term is the government debt effect. When the government increases the consumption tax rate and decreases the wage tax rate, some revenue is postponed to

the next period. This implies that the government needs to issue government bonds. $\frac{(1+r_{t+1}(1-\tau_{kst}))s_t+(1+\mu_{t+1}(1-\tau_{krt}))i_t}{(1+\tau_{ct})^2} N_t$ is the revenue that can be collected in the next period and $\frac{1}{1+r_{t+1}} \frac{(1+r_{t+1}(1-\tau_{kst}))s_t+(1+\mu_{t+1}(1-\tau_{krt}))i_t}{(1+\tau_{ct})^2} N_t$ is the present discounted value of this tax revenue. This is the amount of government bonds that the government needs to issue. When the government issues this amount of debt, it will decrease the net asset level and decrease tax revenue in the second period. This effect is measured by $\frac{\partial R_{t+1}}{\partial(D_t N_t)} \frac{1}{1+r_{t+1}} \frac{(1+r_{t+1}(1-\tau_{kst}))s_t+(1+\mu_{t+1}(1-\tau_{krt}))i_t}{(1+\tau_{ct})^2} N_t$. Note that although individuals increase investment in risky assets and safe assets, this is insufficient to offset the effect of an increase in government bonds.¹¹ Thus, the fourth term measures the cost of shifting tax revenue from the current period to the next period through an increase in the consumption tax rate and a decrease in the wage tax rate. Equation (15) shows that the optimal consumption tax rate is determined to balance the benefit and cost of increasing the consumption tax rate.¹²

3 Conclusions

When the rate of return from risky investments is idiosyncratic and the capital market is incomplete, the expected rate of return from a risky investment is higher than that from a safe investment. However, from a macroeconomics point of view, there is no aggregate risk in the risky investment in the presence of idiosyncratic risks. Thus, from a social point of view, it is desirable to increase investment in risky assets and simultaneously enhance risk sharing. The present study showed that the use of consumption tax increases investment in risky assets, enhances risk sharing, and increases economic efficiency. When the factor prices are fixed, the introduction of consumption tax in the presence of wage tax increases economic efficiency, even if the government has instituted capital income taxes initially in the presence of idiosyncratic risks. This improved efficiency comes from the fact that consumption tax will shift the tax burden in later periods and that the timing effect of consumption tax increases investment in risky assets. When the factor prices are endogenous, the optimal consumption tax rate is determined to balance the benefits of risk sharing and increasing investment in safe and risky assets and the costs of issuing government bonds.

One of the key characteristics of the tax system in developed countries is the use of

¹¹Note that in general $\partial R_{t+1}/\partial(D_t N_t)$ is negative.

¹²When the capital market is complete and there is no capital income tax, the sum of the second, third, and fourth terms becomes zero. The first term also becomes zero because $r_{t+1} = \mu_{t+1}$.

consumption tax with wage tax. This study showed that when there is idiosyncratic risk in capital income and the capital market is incomplete, there is a role for consumption tax.

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